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ASSEMBLEABILITY ANALYSIS USING GAPSPACE MODEL FOR 2D MECHANICAL ASSEMBLY

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ABSTRACT

The most fundamental, and perhaps most important, task in tolerance analysis is to test whether or not the components with tolerances are actually able to fit together. This problem doesn't get much explicit attention because it is implicitly included in current tolerance analysis methods. However, for complex assemblies, especially for over-constrained assemblies, the implicit assumptions of assembly may not be valid. This paper proposes an assembly model, called GapSpace, which can capture the necessary and sufficient conditions needed to execute assembleability analysis. Assembleability analyses for both nominal components and those with tolerances are presented using the GapSpace model. Both worst case and statistical analyses are performed. The model and its attendant solution methods are more suitable to GD&T tolerancing than traditional plus/minus tolerancing.

INTRODUCTION

The design of a mechanical assembly can be described as an iterative process. The process involves two repeated phases: bottom-up design and top-down design. In the top-down design phase, the designer may create an overall layout of the assembly based on functionality and environment first, then go down to describe the form and fabrication for components. The bottom up phase is to determine the actual behavior of the design given the geometry, the relative positions and orientations between components [1,2]. Each individual component provides some function and quality information to the whole assembly. The function and behaviors of the assembly are available only when all components are assembled together. As current CAD systems are still developing assembly level design support, .

Tolerances are used to define the allowable limits of geometric variation that are inherent in the manufacturing process [3]. The assignment of geometric tolerances is always a trade-off: a part with tight tolerances is good for assembly, but the cost to manufacture the part is increased. Alternatively, a loose tolerance in one part may make the whole assembly infeasible. The study of the aggregate behaviors of given individual tolerances is referred to as tolerance analysis [4]. But let's return to that first and foremost question for tolerance analysis: *can the component parts be assembled without interference?* The question is often incorrectly answered – or neglected altogether – by designers, as the current tolerance analysis methods for multidimensional assembly are still evolving. Even for nominally sized of components, the assembleability of multidimensional mechanical assemblies are not explored completely.

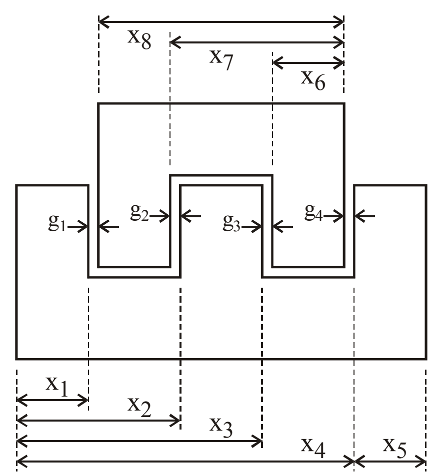


Figure 1: Hinge assembly from Lee [4]

Using an existing paper from the literature [4], we'll look at an assembly, shown in Figure 1, to see how the assembleability might be incorrectly analyzed. The paper tries to find the optimal tolerances based on 4 design functions listed in Equation (1). What is overlooked in this analysis is that another constraint equation (see Equation (2)) may be extracted from the fitting requirement because the smaller part should be able to be plugged into the bigger part in whole. Only after this function (F_5) and functions F_2, F_3 and F_4 in are satisfied can the design for optimal tolerances make sense. We claim that since researchers in this field have difficulty finding the sufficient conditions to test the assembleability, designers may be even more liable to miss them, as they have many other tasks to consider as well [5].

$$\begin{cases} F_1(X) = -x_4 - x_5 + 5.005 \\ F_2(X) = x_2 - x_1 - x_8 + x_7 - 0.0003 \\ F_3(X) = x_7 - x_6 - x_3 + x_2 - 0.001 \\ F_4(X) = x_4 - x_3 - x_6 - 0.0003 \end{cases} \quad (1)$$

$$F_5(X) = x_4 - x_1 - x_8 \geq 0 \quad (2)$$

The difficulty in the previous problem may come from the issue that the example is an over-constrained assembly. The purpose of the assembly is to fix the two parts in their functional direction, but the direction is constrained by several mating surfaces. The tolerance analysis for over-constrained assembly hasn't received much emphasis to date.

The method in this paper is one that may be used to identify over-constrained assembly. The method captures conditions, called fitting conditions (FC), that are used to test the assembleability for 2D mechanical assembly. The method operates on the GapSpace model, which will be explained in detail.

The next section reviews relevant prior work, after which the GapSpace model and its general application are introduced. An example problem is then proposed, and we describe the analysis of the nominal components, followed by worst case and statistical tolerance analyses. Finally, we describe how GapSpace in its present form may be used to interact with various modelers.

REVIEW

The problem of assembleability analysis is tightly connected with tolerance analysis of assembly. Some methods for tolerance analysis may address the problem or include it implicitly in their circumstances.

The linear "stack-up" method is a fundamental technique in one-dimensional assembly tolerance analysis, as described in [6] and others. This analysis requires that assembly components are in contact at mating faces and closed loops are generated through these faces. Every loop is composed by chain of dimensions and at most one single gap. The group led by Ken Chase at Brigham Young University has worked to extend this stack-up analysis to 2- and 3-dimensional assemblies [7,8]. A vector loop-based assembly model is used for these analyses, in which closed vector loops describe the small kinematic adjustments and open vector loops describe critical clearances or other assembly features. A commercial software package based on this work is available and can perform worst-case and statistical analyses. The assembleability analysis for these methods is done by testing if the gap in each loop is greater than zero. But the problem is that some loops may not contain any gap (or may contain *two* gaps, if viewed differently), especially in the over-constrained problem like the example in Figure 1. The loop without any gap is infeasible in real assembly from the view of tolerance analysis.

Turner [9] treated some gaps as the design variables, which are identified by users. Using the combination of model variables, tolerance variables and design variables, tolerance synthesis and analysis can be performed using linear programming, Monte Carlo simulation, and least square methods. Again, the model assumes that only the design variables are the possible gaps and all other features are in contact.

The work of [10] and others is based on kinematic pairings of features, called TTRS. These pairings relate features on the same part or on different parts in an assembly. In order for the kinematic analysis to be performed, this model too requires contact or axis alignment between features. The TTRS work has been implemented in a major CAD package where its primary use is determining the validity of datum reference frames (coordinate systems induced from part features). Extensions of this method to address assembleability analysis are underway, but incomplete.

The work that most closely resembles the GapSpace model is the approach proposed by Mullins and Anderson [11,12]. It uses a graph to represent the whole assembly and find the geometric constraints based on the graph, similar to the graph introduced in the paper. But the method does not use gaps to describe the relation between contacting features, so some difficulties were reported in Mullins' thesis [11]. Its algebraic constraint extraction is quite complicated and no more than one gap is considered for the extraction in each constraint.

The over-constrained assembly is mentioned also Mullins [11]. However, a method to identify over-constrained assemblies is not proposed in the thesis. Kriegel notes that designers sometimes make constraint mistakes such as inadvertent over-constraints [13]. Adams [2] uses screw theory to determine whether or not a set of features over-, under-, or exactly-constrains the location and orientation of the part.

The GapSpace approach for one-dimensional assemblies has been described by the authors [14-16]. After all gaps are identified, an assembly graph is generated. Each fitting condition is a loop in the graph. Operating on these fitting conditions, the assembleability analyses and others in tolerance analysis are easily implemented. The work presented in this paper is an extension of the model to accommodate 2D-assembly for addressing the assembleability problem only.

GAPSPACE MODELING

The section explains the assembly model. Several concepts are introduced first for clarity.

Gap

The gap is the basic concept in the GapSpace model. In other papers [8,9,12], the concept "mating condition" is used for a contacting relationship between different surfaces. The functions of the assembly, including the positions and assembly sequences, can be derived from a specification of the mating conditions. But in this paper, a mating condition is a special gap whose value is required to be zero. The non-zero gap may represent what is called a design variable called by Turner, or simply a pair of part surfaces that may come in contact but are not required to do so.

Three kinds of gaps are considered in this paper, and are shown in Figure 2. A gap (in 2D) is generated by parallel lines (called planar gap), one arc and one line (called cylinder-slider gap), or one vertex and one line (called edge-slider gap). The value of each gap is the shortest distance between the two related features.

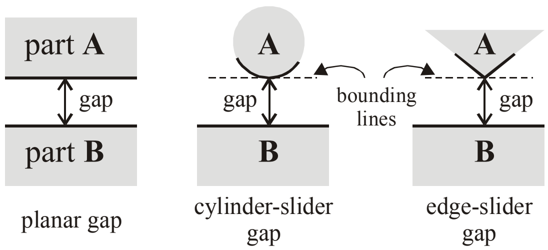


Figure 2: Three types of gaps considered.

When the two features are separated, in contact, or overlapping, the value of the gap is positive, zero, or negative respectively. The direction of a gap is perpendicular to the line of the 'planar' part B shown in Figure 2. For the cylinder slider gap and edge slider gap, we can imagine there is a bounding line representing the extreme extent of the slider which generates a gap like the planar gap.

Constraining Triangle

Each component interacts with the assembly through the gaps that are between features of the different components. A set of gaps is associated with each component, and the identification of the fitting conditions – more on these later – starts with an examination of how an individual component functions in the whole assembly. The basic functional unit in a component is called a Constraining Triangle (CT).

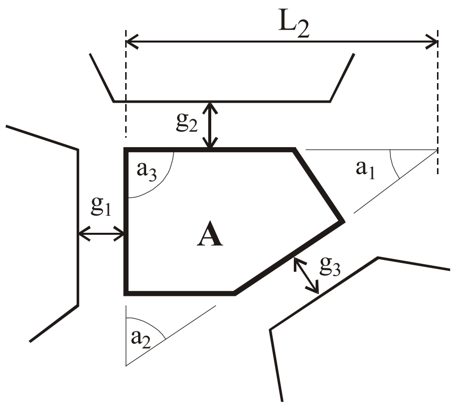


Figure 3: Component A and "the rest of the assembly"

When components are assembled, their mating features constrain the freedoms of their opponents. An individual component may be fully constrained if the gaps related to three features in a constraint triangle are zero (a zero gap means the mating condition is realized.). Figure 3 illustrates an example where the component A is fully constrained when the gaps g_1, g_2 and g_3 are zero. So the constraining triangle inside the component A is defined by the triangle generated by the three features 1, 2 and 3 and their associated gaps g_1, g_2 and g_3.

If any feature is not an edge, its imaginary line is used to generate the constraining triangle. It is called CT_A(g_1, g_2, g_3) for convenience. The special case is that two parallel features with opposing surface normal directions generate a one-dimensional constraining triangle (or a 1D CT), where the 1D CT contains only 2 gaps. The way to identify the effective 2D CT is that the relative angle between any two neighboring gaps should be less than π. The important property of a CT is that the sum in Equation (3) is independent of the relative position of the component in the whole assembly.

$$g_1 \sin(a1) + g_2 \sin(a2) + g_3 \sin(a3) = C \tag{3}$$

Each constraining triangle is associated with a value represented by its geometric parameters and a sign that marks the material side of the CT. The value is called *equivalent constraining triangle length (ECTL)* and calculated by the diameter of the circumscribed circle of the constraining triangle multiplying the sine values of the three angles for 2D CT, or the distance between the two features for 1D CT. The value is independent of the relative position between the component and the remainder of the assembly, and is only dependent on its geometric parameters. The sign of a CT is positive if the material side is inside the triangle (like a pin), and negative if the material is outside the CT.

Assembly graph

The constraint triangle is used to describe constraints on the dimensions of the individual components. The next step is to find a way to represent the relations among these constraints from different components. An assembly graph is used to capture such assembly relationship and induce the assembly-level fitting conditions.

To generate an assembly graph, all gaps of functional importance inside the assembly should be identified, and then the constraining triangles inside each component are determined. There may be more than one constraining triangle for any given component. The assembly graph is generated using following rules:

1. Each constraining triangle is a node in the graph.
2. Every node has 2 or 3 ports that represent their relative gaps. A node with 2 ports or 3 ports is 1D or 2D CT respectively. The ports in the same node are called siblings.
3. Any pair of ports associated with a common gap are connected by an arc if they belong to different components. No connection exists between nodes belonging to the same component. *These arcs are undirected.*

Using these rules, the assembly shown in Figure 4 is translated into the assembly graph in Figure 5, where six CTs (A1, A2, A3, B1, C1, and C2) are identified. Each CT is a triangle in the figure. The example is used for the remainder of this paper and the tolerances for the components are specified in Figure 6.

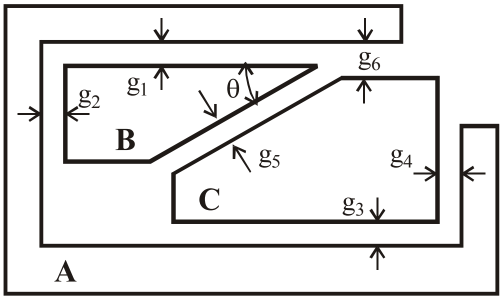


Figure 4: Simple 2D assembly

Fitting Conditions

The purpose of the GapSpace model is to find the fitting conditions (FC) of the assembly. Each fitting condition is a subgraph of the assembly graph. It is an independent sub-assembly that corresponds to a physical condition for assembleability. A fitting condition must satisfy following:

1. It is a subgraph of the assembly graph.
2. The subgraph contains one or more assembly cycles (cycles with some mild restrictions, as described in [14]).
3. If any port in a node is included in the subgraph, other ports in the node should occur in some assembly cycle that is part of the subgraph (i.e. each node in the subgraph must have all ports utilized).
4. The subgraph can't be divided into more subgraphs that satisfy the above 3 conditions.

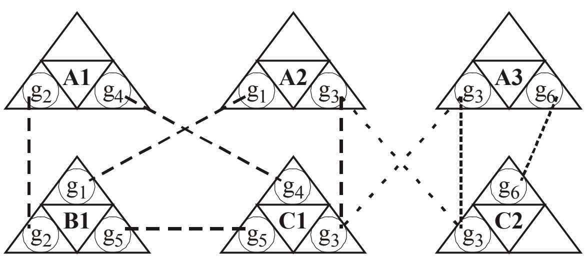


Figure 5: Assembly Graph for Figure 4

Condition 4 above is to prevent our search algorithm from finding redundant fitting conditions. Two fitting conditions are found using a breadth-first search algorithm that will be described in another upcoming paper. One (called FC1) is comprised of A1, A2, B1 and C1. Another (called FC2) includes A3 and C2. FC2 is one-dimensional fitting condition since both CTs inside it are 1D CTs. Inspection of the assembly in Figure 4 shows how these constraints may be interpreted physically.

As each CT functions as basic unit in fitting conditions, the weight of the contribution of each CT in any fitting condition is symbolized by a contribution coefficient (CC). The contribution coefficients inside each FC are relative values, so one of them can always be set to 1 for convenience. In FC1, the ratio of contribution coefficients for CTs (A1, A2, B1, C1) is $\sin\theta : \cos\theta : 1 : 1$. The contribution coefficients in a 1D fitting condition are always 1.

Each fitting condition is associated with 2 representations that are equal to each other: gaps' representation in Equation (4) and geometric parameter representation in Equation (5).

$$FC = \sum_{i=1}^{n} a_i g_i = \sum_{j=1}^{m} CC_j \cdot CT_j / 2 \qquad (4)$$

Where a_i is the weight of gap g_i, CC_i is the contribution coefficient of j^{th} CT, CT_j is the gaps' sum in Equation (3) for j^{th} CT.

$$FC = \sum_{i} CC_i \cdot S_i \cdot ECTL_i \qquad (5)$$

Where i means the ith constraining triangle in the FC, CC_i is the contribution coefficient of i, S_i is the inward/outward sign of i, and $ECTL_i$ is the equivalent CT length of i.

For convenience, all gaps, all fitting conditions, and all part dimensions can be regarded as separate vectors $\vec{G}$, $\vec{F}$, and $\vec{D}$ as shown in Equation (6). Their relationship can be described in Equation (7), which follows from Equations (4) and (5).

$$\begin{aligned} \vec{G} &= (g_1, \cdots, g_i, \cdots, g_n)^T \\ \vec{F} &= (FC_1, \cdots, FC_i, \cdots, FC_m)^T \\ \vec{D} &= (d_1, \cdots, d_i, \cdots, d_k)^T \end{aligned} \qquad (6)$$

$$\vec{F} = M_{GP} \bullet \vec{G} = M_{Dim} \bullet \vec{D} \qquad (7)$$

The matrix M_{GP} is called GapSpace matrix, and M_{GP} is named as dimensional matrix of the GapSpace. When the rank of M_{GP} is smaller than the number of fitting conditions, the assembly must be over-constrained because the dependence in fitting conditions are generated by over-constrained assembly. Therefore, the independent fitting conditions can be calculated based on the matrix.

If each column of M_{Dim} is treated as a vector, e.g. $(\vec{y}_1, \cdots, \vec{y}_i, \cdots, \vec{y}_k)$, the vector $\vec{y}_i$ is called the GapSpace vector of the ith dimension d_i. The vector represents the functions of the dimension to the assembly. The more non-zero elements in $\vec{y}_i$ the more critical the dimension d_i in the assembly. If $\vec{y}_i$ is zero, then d_i doesn't affect the assembleability at all.

The GapSpace model can be used to implement tolerance analysis. In the later sections we will address only the assembleability problem, since there is an equivalent relation between the assembleability and the non-negative fitting conditions.

ASSEMBLEABILITY ANALYSIS FOR NOMINAL COMPONENTS

From the top-down design, the designer may not have such details like manufacturing processes and tolerances, so it is important for designers to know that if the nominal components are able to fit together. It is also meaningful for designers to recognize which dimensions affect the design function or the assembleability. The GapSpace model provides a very good mechanism for answering such questions. The fitting conditions identified from the GapSpace model are the necessary and sufficient conditions to capture the assembleability. In other words, the assembleability is satisfied *if and only if* all fitting conditions are non-negative.

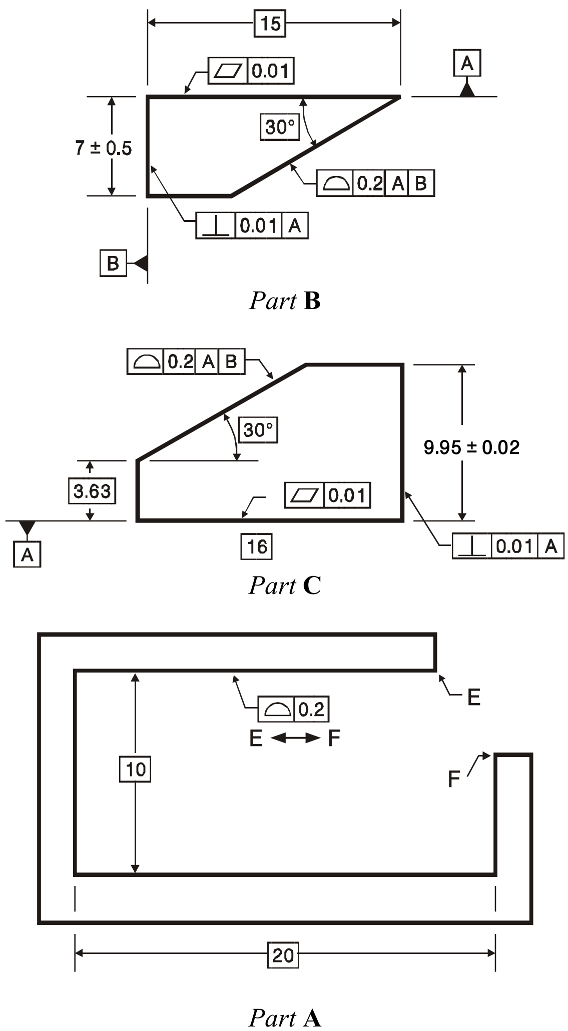


Figure 6: GD&T Specifications for parts.

From Equation 5, the value of each fitting condition is independent from their relative positions of components because the contribution coefficients of CTs are calculated from the geometric parameters only (calculation of the contribution coefficients is covered in an upcoming paper). Consequently the assembleability is also independent from the relative positions of components.

Given the assembly in Figure 4, and its relative dimensions and tolerances in Figure 6, six gaps (g_1 to g_6) are identified and two fitting conditions exist. The gaps' representations of fitting conditions are compressed into Equation (8).

$$FC1(g) = (g_1 + g_3)\cos\theta + (g_2 + g_4)\sin\theta + g_5 \qquad (8)$$
$$FC2(g) = g_1 + g_4$$

Where θ is the angle shown as 30^0 in Figure 6.

The tolerances specified in Figure 6 are consistent with the ASME tolerancing standard [3] and – without excessive explanation – describe zones between parallel lines (or planes) within which the surfaces that comprise the gaps must lie. We can therefore analyze the tolerances by testing the assembleability of the component tolerance zones.

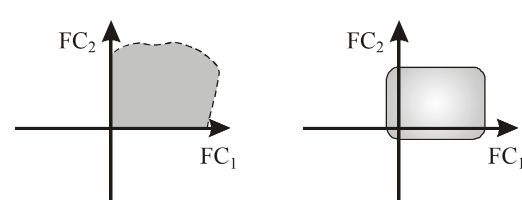
If we do not consider the tolerances defined in Figure 6, the value of the FCs calculated by parametric representations are listed in Equation (9), where the equivalent constraining triangle lengths are calculated from their respective circumscribed circles.

$$\begin{aligned} FC1 &= ECTL(A1)\bullet\sin 30^\circ + ECTL(A2)\bullet\cos 30^\circ \\ &\quad - ECTL(B1) - ECTL(C1) \\ &= 0.017 \\ FC2 &= ECTL(A3) - ECTL(C2) \\ &= 10 - 9.95 = 0.05 \end{aligned} \qquad (9)$$

Because each fitting condition value is each bigger than zero, we can conclude that assembly is possible for the nominal parts. Note that this result is independent of the current actual values of the gap sizes.

WORST CASE ASSEMBLEABILITY ANALYSIS

By assuming each dimension and tolerance in every component to be its maximum or minimum limit, the assembly is tested to check if there is any interference between components. When this method is used, the designer may want to guarantee that the components can be assembled. To illustrate the problem easily, we may discuss the problem in a hyperspace of fitting conditions regarding every independent fitting condition be an axis in the hyperspace. The independence is calculated from GapSpace matrix M_{GP}. Each fitting condition maps a half space separated by a hyperplane since the fitting condition must be non-negative. So the region satisfying the assembleability is the intersection of the half spaces generated by hyperplanes. The hyperspace is called Assembly Space, and the intersection of the hyperplane is called assembly region. These hyperplanes will not all be mutually orthogonal in the case of an over-constrained assembly problem. The assembly region of the example in Figure 4 is shown in Figure 7a.



a. Assembly Region *b. Tolerance Region*

Figure 7: Regions in assembly space.

A tolerance region can be expressed in the assembly space given the tolerances for all related geometric parameters since each fitting condition is associated with some of the components' geometric parameters. The assembleability of the assembly depends on whether the tolerance region is totally included in the assembly region or not. In Figure 7, the assembleability is not satisfied because a portion of the tolerance region is out of the assembly region.

As the functional unit in each fitting condition is the constraining triangle, the variations of the ECTL of each constraining triangle decide the tolerance region. The variation of the ECTL can be thought as the variation of the diameter of its circumscribed circle as shown in Figure 8.

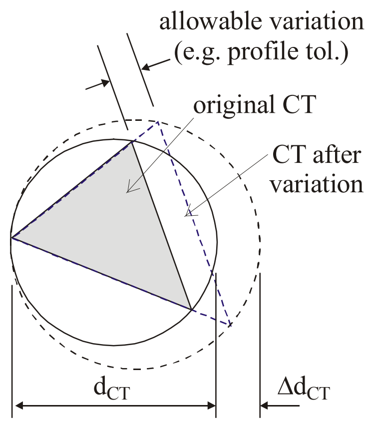


Figure 8: Effects of tolerances on ECTL diameter.

We have found the GapSpace method to be more suitable to GD&T tolerancing than traditional plus/minus tolerancing, because many of the tolerances specified using GD&T result in offsets of the part features, which directly affects the ECTL of the constraining triangle. On the other hand, traditional tolerancing may affect the triangle (angle) itself. The ECTL of the sole constraining triangle of part B in Figure 6 is affected by the profile tolerance. The flatness on datum A and perpendicularity of datum B to A doesn't affect the allowed extremes of the profile tolerance. This is because the variation of the profile is based on a datum reference frame which excludes the variations on the datum.

Traditional tolerancing, as is shown in Figure 9, creates problems for calculating the variations of the ECTL because the variation of the angle changes the value of the contribution coefficients, and a gap may not be enough to describe the relation between two non-parallel features. More research is focused on the issue.

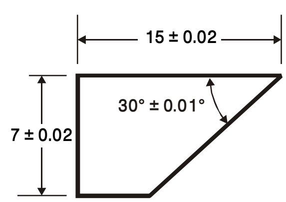


Figure 9: Traditional tolerancing on part B.

Let's use the toleranced example from Figure 6 to see how tolerances affect the tolerance region. The difference between Equations (9) and (10) is that the latter includes the possible variation that could occur while still meeting the tolerances.

$$\begin{aligned} FC1 &= \text{FC1(nominalParts)} + FC1(Tolerances) \\ &= 0.017 \pm (.01 + .01 + .02\sin 30° + .02\cos 30°)/2 \\ &= 0.017 \pm 0.024 \end{aligned} \tag{10}$$

$$\begin{aligned} FC2 &= \text{FC2(nominalParts)} + FC2(Tolerances) \\ &= 0.017 \pm (.02 + .02)/2 \\ &= 0.05 \pm 0.020 \end{aligned}$$

The nominal parts determine a point in the tolerance region and the tolerances decides its range. Since all profiles in the example are bilateral, the tolerance region will be distributed about this nominal point equally. The tolerance region is generated by using a modification of the Minkowski sum [14]. Figure 10 shows the tolerance region of our toleranced assembly.

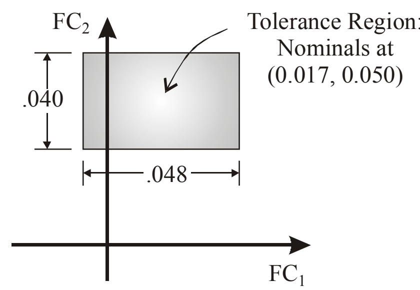


Figure 10: Tolerance region for the assembly.

Since the tolerance region is not fully included in the assembly region (about 20% tolerance region is out), the guarantee of assembleability is not satisfied. There are two possible ways to remedy this. One is to change the nominal dimensions in the model, moving the nominal point to right. However, this may make some gaps too big in some situations, violating other design constraints. Another way is to make the tolerances smaller, but this will increase the cost of making the components.

STATISTICAL ASSEMBLEABILITY ANALYSIS

Statistical analysis can be used by designers and manufacturing personnel to take advantage of statistical averaging over assemblies of parts, allowing the use of less restrictive tolerances in exchange for admitting the small probability of non-assembly. In this section we show how statistical analysis is facilitated by the GapSpace model.

A statistical distribution model for the tolerances must be assumed in order to begin statistical analysis. Much research has been done to compare different models [16-19], but no uniform conclusion is drawn.

If the statistical distribution for each dimension is chosen, a joint probability density function (joint pdf) that represents the probability of each combination of dimension values can be generated. Once the joint pdf has been constructed, the probability of an assembly meeting its assembleability can be calculated by integrating the joint pdf over assembly region as shown in Equation (11).

$$p(assembleability) = \int_{AR}\cdots\int pdf(T_1, T_2, \cdots) dAR \quad (11)$$

Where AR means the assembly region in the assembly space, T_i means the i^{th} tolerance in the assembly.

It's important to note that the joint probability density function may be complex when the correlation between different axes in the assembly space cannot be neglected.

In our example, we assume that each tolerance has a normal distribution from an independent manufacturing process, and the range of each tolerance defined by designers corresponds to ±3σ of the process for making that feature. The distribution of the tolerances over the assembly space is in Equation (12) where the first column μ represents the mean values (nominal parts), and the elements in the diagonal of the matrix Σ are the variances and the elements off the diagonal are their covariance.

$$pdf(AS) = \begin{Bmatrix} FC1 \\ FC2 \end{Bmatrix} \sim nor[\vec{\mu}, \Sigma] = nor\left[\begin{pmatrix} .017 \\ .050 \end{pmatrix}, \begin{pmatrix} 6.7e-5 & 1.7e-5 \\ 1.7e-5 & 8.9e-5 \end{pmatrix}\right] \quad (12)$$

Integrating the pdf over the assembly region, the result is 98%, which means this amount of assemblies will fit together if the components are randomly selected. This is also based on the assumption that the tolerances are normally distributed and a RSS (Root Sum Squares) model is used to combine them.

In the worst case analysis, 20% of the tolerance region is outside of the assembly region, indicating severe measures are necessary to guarantee assembly. But, when performing a statistical analysis, we see the non-assembly percentage is about 2%. This shows the possibility of accepting a few failed assemblies in order to retain looser tolerances.

DISCUSSION

Only three kinds of gaps are considered in the paper as we mentioned earlier. Their common characteristic is that the value of the gap is independent of the actual position along the line perpendicular to the gap. This is the reason the method can't handle the case for a multiple pin-hole assembly, where a scalar gap is not enough to describe the relation between the pin and hole. More work is underway to apply the theories in our method to these more complex assemblies.

When we consider the effects of the tolerances to the equivalent constraint triangle length, GD&T tolerancing is used because it has far fewer ambiguities in the allowable geometry specified. It is a "very clear and concise three dimensional mathematical language for communicating product definition" [20]. The traditional "plus/minus" tolerancing, on the other hand, is fraught with ambiguity, and it isn't clear exactly how to verify the product specification such as that in Figure 9. The GapSpace method also can't predict the effect of this plus/minus variation on the whole assembly. In [14], an example is shown where GD&T tolerancing (using material condition modifiers) has an advantage over traditional tolerancing in that the same tolerance ranges in GD&T allow us to map more tolerance space into the assembly space.

In the GapSpace model described in this paper, calculating the equivalent constraining triangle length automatically is important because it ties the geometric parameters of the parts to the fitting condition. The issue is tightly connected with the system for the representation of dimensions and tolerances conforming to solid modeling systems like Pro/E. [21] proposes a graph structure to represent dimensioning and tolerancing information and validate the dimensioning scheme and tolerance specifications. The structure may be used to induce the equivalent constraining triangle length.

This paper only addresses the problem to test the assembleability for assembly and the over-constraint problem using the GapSpace model. Other tolerance analyses problem can be solved using the same model.

At the present time, the testing of the GapSpace methods in conjunction with various modeling systems is limited to using the CAD package as a keeper of the part geometry and implementing the analysis using a separate process. To bring these methods to bear on general assembly problems requires some additional advances in the theory, and an easy mechanism through which the designer can identify *potential* contacts (i.e. the gaps) instead of *required* contacts. Most CAD systems fix the relative position of the parts in an assembly is using the contact of surfaces or the alignment of centerplanes or axes. To utilize the position-independent analyses described in this paper, this fixing of the parts must be relaxed in order to allow the freedom of assembly that occurs in actual assemblies, where parts are constrained by how they fit, not where they touch. We are extending these methods to accommodate 3D problems and to cover a broader range of gap types than those described here.

CONCLUSION

The GapSpace model uses gaps to simulate the mating relation between features. A graph is generated and a set of fitting conditions is found which captures the necessary and sufficient conditions of assembleability. The assembleability analysis problem is transformed to a test of the relationship between the tolerance region and the assembly region in the assembly space. With two representations of the fitting conditions, assembleability analysis for nominal components and their worst case and statistical tolerances is implemented easily. We are also able to identify over-constrained assemblies based on the relationship between the fitting conditions.

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