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Geometrical product specifications (GPS) –

Bias and uncertainty of dimensional measurements due to thermal influences

Spécification géométrique des produits (GPS) –

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Contents

Foreword

The International Organization for Standardization (ISO) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing international standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare international standards, but in exceptional circumstances a technical committee may propose publication of a technical report of one of the following types:

☐ type 1, when the required support cannot be obtained for publication of an international standard, despite repeated efforts;

☐ type 2, when a subject is still under technical development or where, for any other reason, there is in the immediate future no possibility of an agreement on an international standard;

☐ type 3, when a technical committee has collected data of a different kind from that which is normally published as an international standard ("state-of-the-art", for example).

Technical reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into international standards. Technical reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/DTR 16 015, which is a technical report of type 2, was prepared by the technical committee ISO/TC 213 *Dimensional and geometrical product specifications and verification*

This is the first edition. The terms and definitions used in this technical report are related to existing national standards to maintain continuity.

All Annexes are informative.

Introduction

The reference temperature for industrial length measurements is 20 °C (ISO 1: 1998). ISO 1 does not require that all calibrations, workpiece acceptance tests, or manufacturing be performed at the reference temperature. Uncertainty in temperature measurement and measurement at other than the reference temperature lead to uncertainty and systematic error (bias) in the length measurement result.

The principle addressed by this technical report is that most materials expand or contract when their temperatures are changed. If the temperature at which the measurement is made is the reference temperature, the nominal thermal expansion is zero but uncertainty in the measurement of temperature leads to uncertainty in the measurement result. If length measurements are made at temperatures other than the reference temperature, there will be a resulting differential thermal expansion. This may arise both when the measuring instrument is adjusted, by comparison with a working standard, and when it is used to measure the workpiece.

If the temperatures and the response to thermal changes of the workpiece, the working standard, and the measuring system are known, a correction can be made for differential thermal expansion. It is impossible to know exactly either the temperatures or the response; thus, there will be an uncertainty in the correction and in the measurement result. This technical report shows how to calculate the relevant systematic error (bias) and evaluate the measurement uncertainty.

The resulting standard uncertainty component due to thermal effects must be combined in the normal manner (ISO *Guide to the expression of uncertainty in measurement*) to evaluate the combined standard uncertainty for a calibration or measurement. When necessary, an appropriate decision rule (for example, an acceptable fraction of workpiece tolerance or that embodied in ISO 14253-1) may be invoked to determine the consequence of the thermally-induced dimensional uncertainty on workpiece conformance decisions.

This technical report is a Geometrical Product Specification (GPS) standard and is to be regarded as a global GPS technical report (see ISO/TR 14638).

For more detailed information of the relationship of this International Standard to other standards and to the GPS matrix model see annex A.

ISO/DTR 16 015 is developed in support of ISO 1.

Geometrical product specifications (GPS) – Bias and uncertainty of dimensional measurements due to thermal influences

1 Scope

This international technical report defines procedures for computing appropriate corrections and evaluating uncertainty when dimensional measurements are performed (a) when the mean temperature is the reference temperature; (b) when the mean temperature is not the reference temperature where the mean is taken over time and space; and (c) when the temperature varies with time. In this report, we do not deal with the important case of temperature gradients. Three cases are considered: (1) when the mean temperature is the reference temperature; (2) when the mean temperature is not the reference temperature and the user makes corrections; and (3) when the mean temperature is not the reference temperature and the user makes no corrections.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this draft technical report. At the time of publication, the editions indicated were valid. Also, standards are subject to revision, and parties to agreements based on this technical report are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below.

ISO 1: 1998, *Standard reference temperature for industrial length measurements*.

ISO 3534 – 1: 1993, *Statistics – Vocabulary and symbols –* Part 1: *Probability and general statistical terms.*

ISO 3534 – 2: 1993, *Statistics – Vocabulary and symbols –* Part 2: *Statistical quality control.*

ISO 14253 – 1: 1998, G*eometric Product Specifications (GPS) - Inspection by measurement of workpieces and measuring equipment –* Part 1: *Decision rules for proving conformance or non-conformance with specifications.*

ISO/DTR 14253 – 2: 1998 (E), *Geometric Product Specifications (GPS) - Inspection by measurement of workpieces and measuring equipment -* Part 2: *Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification.*

ISO VIM:1993, *International vocabulary of basic and general terms in metrology*.

ISO/TAG4:1995, *Guide to the expression of uncertainty in measurement*.

3 Terms and Definitions

3.1 Terms concerning coefficient of thermal expansion

3.1.1

coefficient of thermal expansion

α

ratio of the fractional change of length to the change in temperature

NOTES: 1 In general, this coefficient is a function of temperature. However, for the purpose of this technical report, a temperature range averaged value of the coefficient is used according to:

$$\alpha(20,T) = \frac{L_T - L_{20}}{L_{20}(T-20)} \qquad (1)$$

where the temperature range is relative to 20 °C per ISO 1.

2 The term, "coefficient of thermal expansion," will be denoted α, with a subscript "w" (i.e., α_w) for the workpiece and a subscript "s" (i.e., α_s) for the working standard.

3 The term “coefficient of thermal expansion” can describe a non-homogeneous body such as a composite working standard or a workpiece that is an assembly of different materials. No distinction is made in this technical report between coefficients of expansion for homogeneous and composite working standards or workpieces.

3.1.2

measured coefficient of thermal expansion

α_m

experimentally-determined coefficient of thermal expansion of a specific individual object

NOTE: The measured coefficient of thermal expansion may be obtained from the calibration service of an accredited measurement institute or from properly performed experiments. It is denoted α_{mw} for the workpiece and α_{ms} for the working standard.

3.1.3

nominal coefficient of thermal expansion

α_n

approximate value for the coefficient of thermal expansion over a range of temperature from 20 °C to T

[ISO VIM: 1993, Clause 5.3]

NOTE: The nominal coefficient of thermal expansion is denoted α_{nw} for the workpiece and α_{ns} for the working standard. Estimated values for α_{nw} and α_{ns} may be obtained from experiments on like objects, or from published data.

3.1.4

uncertainty of coefficient of thermal expansion

$u_{(\alpha)}$

dispersion of the values that could reasonably be attributed to the coefficient of thermal expansion

NOTES: 1 The uncertainty of coefficient of thermal expansion is denoted $u(\alpha_w)$ for the workpiece and $u(\alpha_s)$ for the working standard. This value, like that of α itself, must either be an estimate, based on an assumed probability distribution, or be the result of experiments to measure α.

2 The uncertainty for α_n is usually much larger than for α_m.

3.2 Terms concerning thermal expansion

3.2.1

thermal expansion

Δ_E

change in length of a workpiece or working standard in response to a temperature change

3.2.1.1

thermal expansion based on nominal or measured coefficients of thermal expansion

Δ_{nE} or Δ_{mE}

estimate of the thermal expansion of an object from 20 °C to its average temperature at the time of measurement based on nominal or measured coefficients of thermal expansion:

$$\Delta_{nE} = \alpha_n L(T - 20) = \alpha_n L\theta$$

$$\Delta_{mE} = \alpha_m L(T - 20) = \alpha_m L\theta$$

$$\Delta_E = \Delta_{nE} \text{ or } \Delta_{mE} \tag{2}$$

NOTE: An additional subscript, $_w$ or $_s$, is added to each symbol to specify values for a workpiece or a working standard.

3.2.2

corrected length

L_c

measured length adjusted for the computed thermal expansion based on either the measured or nominal thermal expansion

NOTE: The corrected length can be calculated to

$$L_c = L_m - \Delta_{mE} \quad \text{or}$$

$$L_c = L_m - \Delta_{nE} \tag{3}$$

3.2.3

differential thermal expansion

difference between the changes in the lengths of the workpiece and the working standard in response to temperature changes from 20 °C to their temperatures at the times of the measurements

NOTE: If not corrected, differential thermal expansion may cause a systematic error in a measurement result.

3.2.3.1

differential thermal expansion based on nominal or measured coefficients of thermal expansion

Δ_{nDE} or Δ_{mDE}

difference between the thermal expansion of the workpiece and the working standard(s)

NOTE:

$$\Delta_{nDE} = \Delta_{nEw} - \Delta_{nEs} \qquad \text{or}$$

$$\Delta_{mDE} = \Delta_{mEw} - \Delta_{mEs} \tag{4}$$

3.2.4

uncertainty of thermal expansion due to uncertainty in α

$u_E(L)$

uncertainty in the thermal expansion arising from uncertainty in the coefficient of thermal expansion

$$u_E(L) = L\theta u(\alpha) \tag{5}$$

NOTE: An additional subscript, $_w$ or $_s$, is added to each symbol to specify values for a workpiece or a working standard.

3.2.5

uncertainty in differential thermal expansion due to uncertainties in α_w and α_s

$u_{DE}(L)$

square root of the sum of the squares of the uncertainties of thermal expansion of the workpiece and working standard due to uncertainties in their coefficients of expansion

NOTES: 1 Assuming that the coefficients of thermal expansion of the workpiece and the working standard are uncorrelated, the value of $u_{DE}(L)$ is given by:

$$u_{DE}(L) = \sqrt{u_E^2(L_w) + u_E^2(L_s)} \tag{6}$$

$$= \sqrt{\theta_w^2 L_w^2 u^2(\alpha_w) + \theta_s^2 L_s^2 u^2(\alpha_s)}.$$

2 The estimates of the coefficients of thermal expansion of the workpiece and working standard can be assumed to be uncorrelated when they are obtained from different sources.

3 When nominal values are used to estimate the coefficients of thermal expansion, this quantity has historically been called the “uncertainty of nominal differential expansion” or “UNDE”.

3.3 Dimensional consequences of environmental temperature variation

3.3.1

dimensional thermal response

amplitude of length variation of an object as a thermal response to the magnitude and time-dependency of an environmental temperature fluctuaation

3.3.2

thermal response time (soak-out time)

period of time after which an object is in equilibrium with a change in its thermal environment

NOTE: When a change in environment is experienced, such as occurs when an object is transported from one room to another, there will be some period of time over which the object equilibrates, within specified limits, to its new environment. This response time [ISO VIM:1993 Clause 5.17], following the change in environment until the object may be considered to be influenced only by the temperature of the new environment, is called the thermal response time or the soak-out time.

3.3.3 differential thermal response

length difference between any two objects measured simultaneously in an environment with varying temperature and caused solely by the variation in temperature of the objects with time

3.3.4

dimensional variation due to environmental temperature variation

E_{TVE}

estimate of the possible length measurement variation induced solely by deviation of the environment from average conditions over a time interval equivalent to the adjustment cycle time

NOTE: The dimensional variation due to environmental temperature variation E_{TVE} is usually determined from the results of two drift tests (Clause 3.4.7), one of the working standard and comparator and the other of the workpiece and the comparator. However, drift tests performed at a single position cannot reveal all thermally-induced errors, particularly where spatial temperature gradients exist. It is the responsibility of the instrument user to ensure that such effects are detected and the appropriate uncertainties added.

3.3.5

uncertainties of workpiece and working standard temperatures

$u(\theta_w)$ and $u(\theta_s)$

time-averaged temperature uncertainties of the workpiece and working standard

NOTES: 1 These uncertainties arise from the calibration of the thermometer (Clause 3.4.1), thermometer mounting procedures, and instrumental variations.

2 The time over which the temperature is averaged is usually the period of a measurement cycle.

3.3.6

length uncertainty due to temperature measurement

$u_{TM}(L)$

uncertainty in a measured length due to uncertainty in the measurement of the temperature at which the length measurement was made

NOTES: 1 In the case where there is a workpiece and a working standard, it is given by:

$$u_{TM}(L) = \sqrt{\alpha_w^2 L_w^2 u^2(\theta_w) + \alpha_s^2 L_s^2 u^2(\theta_s)} \tag{7}$$

where $u(\theta_w)$ and $u(\theta_s)$ are the uncertainties in temperature measurement of the workpiece and the working standard (see Clause 3.3.5). The temperature measurements of the workpiece and the working standard are assumed to be uncorrelated, an assumption which is valid if the two temperatures are measured using two different thermometers, each of which is traceable to the International Temperature Scale (ITS-90) through different calibration processes.

2 In cases where the standard is a laser interferometer, the coefficient relating wavelength to the temperature of the propagation medium is used for the coefficient of expansion of the working standard. In the case where the medium is "standard" air, the value for α_{ns} is 0,93 x 10^{-6}/°C.

3.3.7

dimensional uncertainty due to environmental temperature variation

$u_{ETVE}(L)$

uncertainty in the estimate of the possible length measurement variation induced solely by deviation of the environment from average conditions over a time interval equivalent to the adjustment cycle time

NOTES: 1 The dimensional uncertainty due to envonmental temperature variation $u_{ETVE}(L)$, arises from the range of the drift of the instrument/working standard/workpiece system over times equivalent to the adjustment cycle time. It is established from measurements or estimates of dimensional variation due to environmental temperature variation, E_{TVE}.

2 In computing $u_{ETVE}(L)$, note that the maximum error is the range observed in the drift test, depending on whether the mastering was done at the lowest temperature and the measuring at the highest temperature, or vice versa. The probability density function may be "U-shaped," as for example for a sinusoidally varying temperature. However, the probability that the mastering and measuring are exactly displaced in time by the period of the sinusoidally varying temperature is low. The mastering-measuring process involves taking pairs of points from the U-shaped probability distribution function and then differencing them. The consequence of this procedure is that the measurement process has a different probability distribution function from "U-shaped". The default is to use a uniform distribution [See Clause 5.3 and Eq. (13)].

3.4 Measuring instruments, measuring procedures, and metrology

3.4.1

thermometer

instrument used to measure temperatures of the workpiece, the working standard, the environment, and the comparator

3.4.2

comparator

measuring device used to perform a comparison of a workpiece and a working standard

NOTE: A comparator can be a simple short-range indicating device, such as a gage block comparator, or a complex comparator such as a coordinate measuring machine.

3.4.3

working standard

standard that is used routinely to calibrate or check material measures, measuring instruments or reference materials

[ISO VIM:1994 Clause 6.7]

NOTE: For procedures which comply with the recommendations of this technical report, such working standards are dimensional. The standard may be in the form of the wavelength of light, a gage block, a line standard, a lead screw, etc.

3.4.4

adjustment

operation of bringing a measuring instrument into a state of performance suitable for its use

[ISO VIM: 1993, Clause 4.30]

NOTE: The action of nulling or setting a comparator with a working standard is an example of adjustment.

3.4.5

adjustment cycle time

period between successive adjustments of a comparator

3.4.6

drift test

experiment conducted to determine the change in the measured value given by a measurement system under normal operating conditions over a duration of time

[ISO VIM:1993, Clause 5.16]

3.4.7

tolerance

TOL

difference between the upper and lower tolerance limits

[ISO 3543-2:1993, Clause 1.4.4]

NOTE: In the context of this ISO Standard the abbreviation *TOL* is used instead of the standardized abbreviation *T* to avoid any confusion with the abbreviation of *T* for temperature.

3.4.8

target uncertainty

U_T

uncertainty determined as the optimum for the measuring task

[ISO/TR 14253-2]

3.4.9

systematic error of result

component of the error which, in the course of a number of test results for the same characteristic, remains constant or varies in a predictable way

[ISO 3534-1:1993, Clause 3.10]

3.4.10

bias

difference between the expectation of the test results and an accepted reference value

[ISO 3534-1:1993, Clause 3.13]

NOTE: For the purpose of this technical report, bias is taken to mean the systematic error due to differential thermal expansion.

3.5 Dimensional quantities related to thermal effects

3.5.1

Standard uncertainty component due to thermal effects

$u_{cT}(L)$

standard uncertainty component due to thermal effects for a length measurement made at a temperature other than 20 °C in a changing environment:

$$u_{cT}(L) = \sqrt{u_{ETVE}^2(L) + u_{DE}^2(L) + u_{TM}^2(L)} \tag{8}$$

NOTES: A simplified form arises in the case where: $\alpha_S = \alpha_{nS} = \alpha_w = \alpha$ with a standard uncertainty $u(\alpha)$; $\theta_w = \theta_s = \theta$ with a standard uncertainty $u(\theta)$; and where the nominal lengths $L_w = L_s = L$. In this case, $\Delta_{nDE} = 0$ and

$$u_{cT}(L) = \sqrt{u_{ETVE}^2(L) + 2\theta^2 L^2 u^2(\alpha) + 2\alpha^2 L^2 u^2(\theta)}.$$

3.5.2

thermal error

TE

estimate of the maximum error that might reasonably be expected to occur if a length measurement is not corrected for nominal differential expansion:

$$TE = [|\Delta_{nDE}| + 2u_{cT}(L)] \tag{9}$$

NOTE: If dimensional measurements are made at other than 20 °C and in the special case that corrections are not made for the nominal differential thermal expansion between working standard and workpiece, then in order to follow the recommendations of this technical report, the thermal error shall be evaluated and reported.

3.5.3

thermal error index

TEI

fraction of the Tolerance that is attributable to the Thermal Error:

$$TEI = (2 \times TE/TOL) \times 100\% \tag{10}$$

NOTES: 1) In the case where a target uncertainty (U_T) exists, the thermal error index is computed using Eq. (10), with TOL replaced by U_T.

2) The *TEI* is a component of uncertainty management; large values of the *TEI* may require that a measurement result be corrected for differential thermal expansion in order to prove workpiece conformance.

4 Symbols (and abbreviations)

The reference temperature is 20 °C. All temperatures are assumed to be in Celsius.

4.1 Symbol conventions

The majority of the symbols used in this technical report are based on the following convention:

α_{ij} coefficient of thermal expansion where:

- $i = m$ or n, where m is measured and n is nominal
- $j =$ w or s, where w represents the workpiece and s represents the working standard

$\Delta_{(sub)DE}$ difference between thermally-induced expansions of the workpiece and working standard

where (sub) may contain:

- m or n, where m is measured and n is nominal

$\Delta_{(sub)E}$ thermally-induced expansions where (sub) may contain:

- m or n, where m is measured and n is nominal
- w or s, where w represents the workpiece and s represents the working standard

L_j length where j may be:

- m = measured, uncorrected for temperature
- n = nominal
- c = corrected for nominal expansion
- T = at temperature T
- w or s, where w represents the workpiece and s represents the working standard

T_j temperature where:

- $j =$ w or s, where w represents the workpiece and s represents the working standard

θ_j temperature difference (T_j -20 °C) where:

- $j =$ w or s, where w represents the workpiece and s represents the working standard

u (symbol) uncertainty of the quantity associated with the symbol given in parentheses

4.2 List of symbols

α	coefficient of thermal expansion
α_m	measured coefficient of thermal expansion
α_{mw}	measured coefficient of thermal expansion of workpiece
α_{ms}	measured coefficient of thermal expansion of working standard
α_n	nominal coefficient of thermal expansion
α_{nw}	nominal coefficient of thermal expansion of workpiece
α_{ns}	nominal coefficient of thermal expansion of working standard
α_w	average coefficient of thermal expansion of workpiece
α_s	average coefficient of thermal expansion of working standard
d	reading of instrument (Annex D)
Δ_{DE}	differential expansion between a workpiece and the working standard
Δ_E	thermal expansion
Δ_{mDE}	differential expansion between a workpiece and the working standard based on measured coefficients of expansion
Δ_{mE}	thermal expansion based on a measured coefficient of thermal expansion
Δ_{nDE}	nominal differential thermal expansion between a workpiece and the working standard based on nominal coefficients of thermal expansion
Δ_{nE}	nominal thermal expansion based on a nominal coefficient of thermal expansion
E_{TVE}	dimensional variation due to environmental temperature variation
k	coverage factor (Guide to the Expression of Uncertainty In Measurement)
L	dimension
L_c	dimension of an object measured at temperature, T, and corrected for nominal thermal expansion
L_i	dimension of a measuring instrument or comparator measured at temperature, T,and corrected for nominal thermal expansion
L_m	dimension of an object measured at temperature, T, and not corrected for nominal thermal expansion
L_n	nominal dimension of a workpiece
L_w	dimension of workpiece at 20 °C
L_s	dimension of reference or working standard at 20 °C

L_T	dimension of an object at temperature, T
t	time
T_w	temperature of workpiece
T_s	temperature of working standard
TE	thermal error
TEI	thermal error index
TOL	dimensional tolerance
τ	time constant of a physical quantity
θ_w	temperature difference (T_w-20 °C) of workpiece from 20 °C
θ_s	temperature difference (T_s -20 °C) of standard from 20 °C
$u(\alpha)$	uncertainty of coefficient of thermal expansion
$u(\alpha_i)$	uncertainty of the coefficient of thermal expansion of measuring instrument
$u(\alpha_w)$	uncertainty of the coefficient of thermal expansion of workpiece
$u(\alpha_s)$	uncertainty of the coefficient of thermal expansion of working standard
$u(\theta_i)$	uncertainty of temperature of measuring instrument
$u(\theta_w)$	uncertainty of temperature of workpiece
$u(\theta_s)$	uncertainty of temperature of working standard
$u_{cT}(L)$	standard uncertainty component due to thermal effects
$u_{DE}(L)$	uncertainty of differential thermal expansion due to uncertainty in α_s and α_w
$u_E(L)$	uncertainty of thermal expansion
$u_{ETVE}(L)$	dimensional uncertainty due to the environmental temperature variation
$u_{TM}(L)$	dimensional uncertainty due to temperature measurement
U_T	target uncertainty

5 Procedure

If a dimensional measurement is performed at a temperature other than the reference temperature of 20°C, there will be a systematic effect due to differential thermal expansion between the workpiece and the working standard. This effect will cause a systematic error if the measurement result is not corrected. The decision whether or not to correct a measurement result for the effect of differential thermal expansion is a management decision based on costs and risks. In any case, it is necessary to evaluate the thermally-related components of the measurement uncertainty budget. A general procedure is as follows:

a) Measure the length of the workpiece, yielding an uncorrected result L_m.

b) Evaluate the relevant temperatures and their uncertainties (see Clause 5.1).

c) Evaluate the thermal expansion coefficients and their uncertainties (see Clause 5.2).

d) Evaluate the dimensional uncertainty due to environmental temperature variation, $u_{ETVE}(L)$ (see Clause 5.3).

e) Calculate the standard uncertainty component due to thermal effects, $u_{cT}(L)$ (see Clause 5.4).

f) Calculate the differential thermal expansion, Δ_{DE} (see Clause 3.2.3).

g) If the measured length is to be corrected, perform the correction (see Clause 3.2.1.1).

h) and report the corrected length, L_c, and the standard uncertainty component due to thermal effects, $u_{cT}(L)$.

i) If no correction is to be performed, report the uncorrected result, L_m, the differential thermal expansion (systematic error), Δ_{DE}, and the standard uncertainty component due to thermal effects, $u_{cT}(L)$. If there is a tolerance or a target uncertainty, also compute and report the thermal error index, *TEI* (see Clause 3.5.3). This procedure is illustrated in Annex C Flow Chart. It should be recognized that thermal influences are only some of the effects that must be taken into account in the evaluation of the combined standard uncertainty of a dimensional measurement. Once the standard uncertainty component due to thermal effects has been computed, it may be:

a) Combined in quadrature with the other standard uncertainties, following the recommendations of the ISO *Guide to the Expression of Uncertainty in Measurement*, in order to evaluate the combined standard uncertainty of the measurement ; or

b) Combined in quadrature with other standard uncertainties in order to evaluate an estimated uncertainty that can be compared with a target uncertainty in support of a business decision.

5.1.1 Evaluate uncertainties of workpiece and working standard temperatures (see Clause 3.3.5)

These uncertainties can be evaluated by various methods. For example,

a) The temperature of the body may be measured and the uncertainty computed from this measurement in accordance with ISO guidelines.

b) The evaluation may be based on the distribution found among results of measurements conducted on a number of like objects, using the same thermometers and the same procedures.

c) The evaluation may be based on the distribution found in published data regarding the use of such thermometers and specific procedures.

d) The evaluation may be based upon judgment regarding the range of possible error. For the purposes of this technical report and in the absence of other information, we recommend evaluating the temperature measurement uncertainty by using assumed rectangular distributions for the relevant temperatures (see Annex E for an example).

For case (d), where the knowledge of the temperatures is represented by a rectangular (uniform) distribution, the standard uncertainty in temperature measurement is given by:

$$u(\theta) = \frac{a^+ - a^-}{2\sqrt{3}}, \tag{11}$$

where a^+ and a^- are the upper and lower limits of the rectangular distribution, respectively.

5.2 Evaluate uncertainties of coefficients of thermal expansion, $u_{(\alpha)}$ (see Clause 3.1.3)

Various methods can be used to make this evaluation. For example,

a) The thermal expansion of the bodies may be measured and the uncertainties associated with these measurements adopted.

b) The evaluation may be based on the distribution found among results of actual experiments conducted on a number of like objects.

c) The evaluation may be based on the distribution found among published data.

d) The evaluation may be based upon judgment regarding the range of possible error. For the purposes of this technical report and in the absence of other information, we recommend estimating the uncertainty in the coefficient of thermal expansion using an assumed rectangular distribution to represent knowledge of the quantity (see Annex F for an example).

For case (d), the standard uncertainty in the coefficient of thermal expansion is given by:

$$u(\alpha) = \frac{a^+ - a^-}{2\sqrt{3}}, \tag{12}$$

where a^+ and a^- are the upper and lower limits of the rectangular distribution, respectively.

5.3 Evaluate dimensional uncertainty due to environmental temperature variation, u_{ETVE} (see Clause 3.3.7 and Annex C)

The dimensional uncertainty due to environmental temperature variation u_{ETVE} is obtained from E_{TVE}, i.e., from a drift test using one of two procedures:

a) Measure the E_{TVE} for a given measurement system to obtain a distribution of E_{TVE} over the required measurement time. This distribution could then be analyzed according to its actual form and a standard uncertainty computed following accepted statistical procedures.

b) Assume that the possible values for the environmental error are uniformly distributed within the range of the E_{TVE} obtained from a single test for a given measurement. The resulting standard uncertainty, $u_{ETVE}(L)$, is then given by:

$$u_{ETVE}(L) = E_{TVE} / 2\sqrt{3} \tag{13}$$

5.4 Calculate standard uncertainty component due to thermal effects, $u_{cT}(L)$

The standard uncertainty component due to thermal effects is evaluated using the following procedure :

a) Evaluate the dimensional uncertainty due to environmental temperature variation, $u_{ETVE}(L)$ (see Clause 5.3).

b) Evaluate the uncertainty in the differential thermal expansion due to uncertainties in the coefficients of thermal expansion, $u_{DE}(L)$ (see Clause 3.2.5).

c) Evaluate the uncertainty in the differential thermal expansion due to uncertainties in the temperatures, $u_{TM}(L)$ (see Clause 3.3.6).

d) Evaluate the standard uncertainty component due to thermal effects for a length measurement made at a temperature other than 20 °C in a changing environment, $u_{cT}(L)$ (see Clause 3.5.1).

Annex A

Relationship to the GPS matrix model

For full details about the GPS matrix model see ISO/TR 14638.

A.1 Information about this international technical report and its use

This technical report is a “secondary guide” to the computation of bias, corrections and uncertainty due to thermal influences on dimensional measurements, based on the Guide to the Expression of Uncertainty in Measurement (ISO Guide 1995).

A.2 Position in the GPS matrix model

This technical report is a global GPS technical report, that influences chain links 4, 5 and 6 in all chains of standards in the GPS matrix structure, as graphically illustrated in Figure A.1.

Fundamental GPS standards

Global GPS standards

General GPS matrix

Chain link number	1	2	3	4	5	6
Size				x	x	x
Distance				x	x	x
Radius				x	x	x
Angle				x	x	x
Form of line independent of datum				x	x	x
Form of line dependent of datum				x	x	x
Form of surface independent of datum				x	x	x
Form of surface dependent of datum				x	x	x
Orientation				x	x	x
Location				x	x	x
Circular run-out				x	x	x
Total run-out				x	x	x
Datums				x	x	x
Roughness profile				x	x	x
Waviness profile				x	x	x
Primary profile				x	x	x
Surface imperfections				x	x	x
Edges				x	x	x

Figure A.1 —

A.3 Related international standards

The related international standards are those of the chains of standards indicated in figure A.1.

Annex B

Bibliography

[1] ISO/TR 14638:1995, *Geometrical product specifications (GPS) - Master plan*

[2] ANSI/ASME, "*Temperature and Humidity Environment for Dimensional Measurement*," American National Standard B89.6.2 -1973, Am. Soc. of Mechanical Engineers, New York, 1973 (R1979, R1988)

[3] International Organization for Standardization (ISO*), "International Vocabulary of Basic and General Terms in Metrology*", ISBN 92-67-01075-1,Geneva (1993)

[4] International Organization for Standardization (ISO) *"Guide to the Expression of Uncertainty in Measurement*," ISBN 92-67-10188-9, Geneva (1995)

[5] ISO 14253-1: 1998, *Geometrical Product Specifications (GPS) - Inspection by measurement of workpieces and measuring equipment: - Part 1: Decision rules for proving conformance or non-conformance with specification*

[6] ISO/DTR 14253-2:1998(E), *Geometrical Product Specifications (GPS) - Inspection by measurement of workpieces and measuring equipment: - Part 2: Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification*, ISO, Geneva (1998)

[7] ISO 3534-1 : 1993, *Statistics-Vocabulary and symbols – Part 1 : Probability and general statistical terms*

Annex C

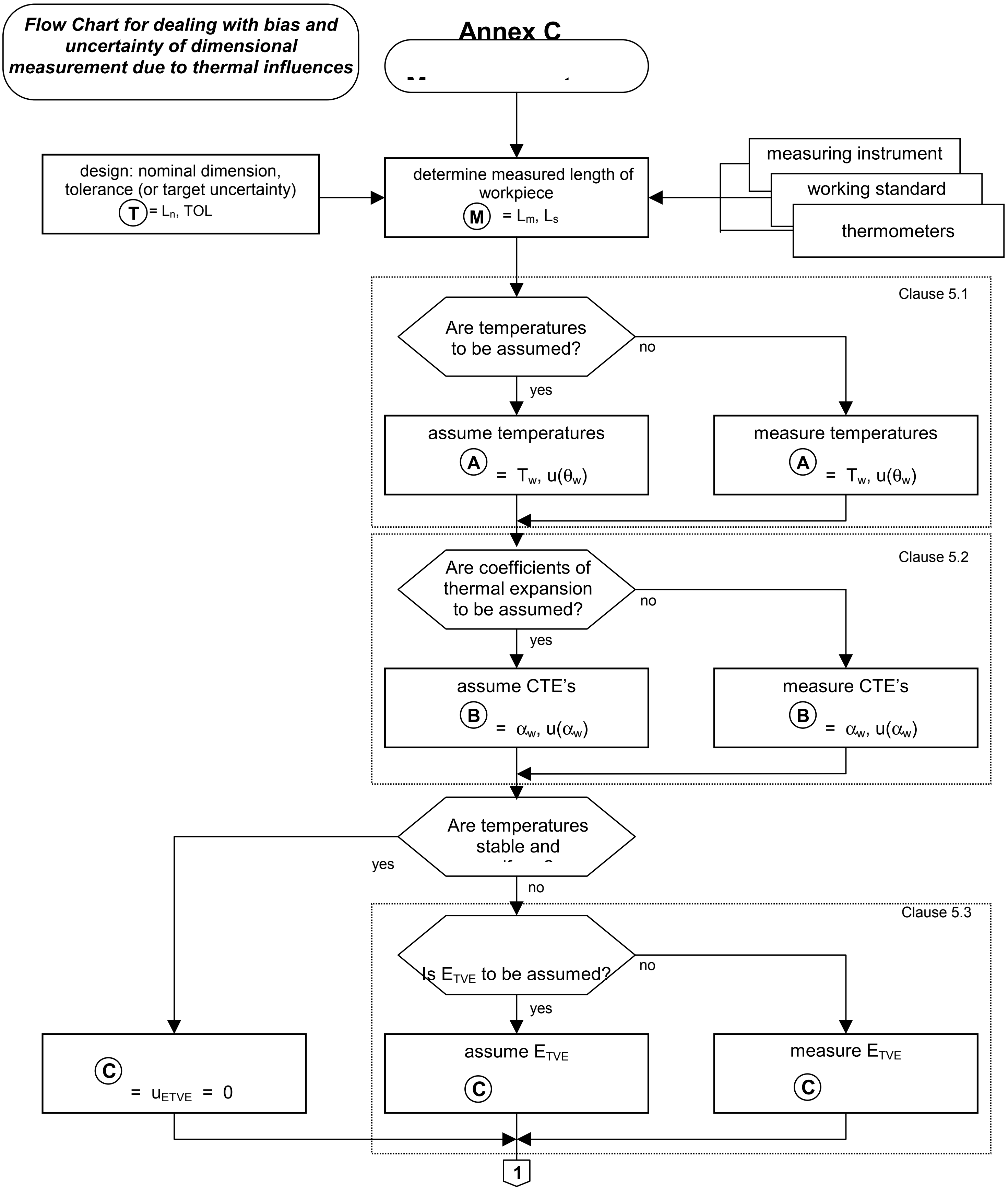

Figure 1a.

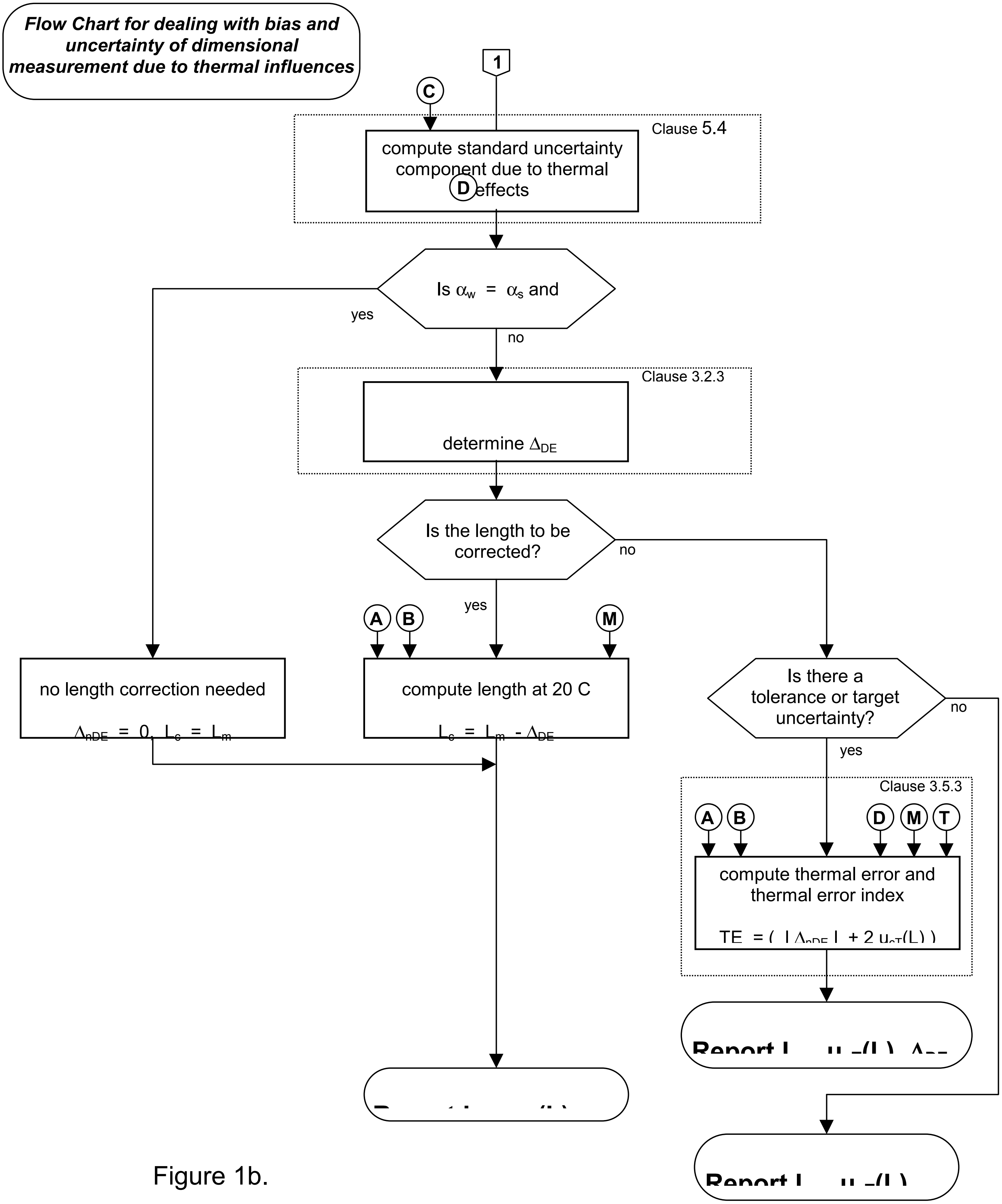
Flow Chart for dealing with bias and uncertainty of dimensional measurement due to thermal influences
1
C
Clause 5.4
compute standard uncertainty component due to thermal effects
D
Is αw = αs and
yes
no
Clause 3.2.3
determine ΔDE
Is the length to be corrected?
no
yes
A
B
M
no length correction needed
ΔnDE = 0, Lc = Lm
compute length at 20 C
Lc = Lm - ΔDE
Is there a tolerance or target uncertainty?
no
yes
Clause 3.5.3
A
B
D
M
T
compute thermal error and thermal error index
TE = (| ΔnDE | + 2 ucT(L))
Report L uc(L) ΔDE
Report L uc(L)

Figure 1b.

Annex D

Index of definitions

Annex E

Advisory information pertaining to temperature environment for dimensional measurement

In this Annex, it is assumed that the measuring equipment and the thermal environment exist, and that normal or expected operating conditions are in force. The object of the discussion is to describe the manner in which one determines the extent of bias and uncertainty resulting from non-ideal temperature conditions.

The ideas and methods described are those found in fairly common usage by metrologists everywhere, but, for the first time, these ideas and methods are unified and formally presented. Some of the concepts presented may at first appear strange and unrelated to previous experience. The 3-element system concept, for example, will probably fall in this category. However, with a little patient study, the concept will be seen to correspond to common thoughts, and its utility in a disciplined investigation will become clear.

The other concepts that may appear to be new are those of the uncertainty of the coefficient of thermal expansion and the uncertainty of the temperature measurement.

These concepts are examined and reduced to a practical procedure (see Sections E1-E3).

Section E4 is devoted to explaining the thermal error index and its use.

E1 Estimation of consequences of mean environmental temperatures other than 20 °C

E1.1 Length measurements. The consequences of temperature variation are easily obtained by means of equations that give the nominal differential thermal expansion in terms of the nominal thermal expansions of the workpiece and the master or working standard [see Equations (2) and (4) in Clause 3.2]

$$\Delta_{nDE} = \Delta_{nEw} - \Delta_{nEs} \tag{E.1}$$

and

$$\Delta_{nE} = \alpha L(T - 20°C) \tag{E.2}$$

Combining these equations, we get

$$\begin{aligned}\Delta_{nDE} &= \alpha_w L_w (T_w - 20°C) - \alpha_s L_s (T_s - 20°C) \\ &= L[\alpha_w (T_w - 20°C) - \alpha_s (T_s - 20°C)]\end{aligned} \tag{E.3}$$

where we assume that $L \approx L_s \approx L_w$.

Assuming that the workpiece and working standard both are at the mean temperature, $T_w = T_s = T_{mean}$ (the only reasonable assumption unless thermometers are attached to both the workpiece and working standard), we see that the bias is reduced to insignificance if the coefficients of thermal expansion approach equality. This is true even with a large deviation of the mean environmental temperature from 20 °C.

Because the great majority of manufactured workpieces and gages are of ferrous materials having similar coefficients of thermal expansion, many industries, particularly those where tolerances are in tenths of millimetres or the workpieces are small, have successfully functioned without concern over the effect of mean environmental temperature on manufacturing accuracy. In many such situations, an arbitrary insistence on 20 °C temperature control leads to an unjustified increase in the cost of manufacture.

As tolerances become tighter, as the workpieces become bigger, and as the materials of workpieces and working standards become more dissimilar, the consequences of mean environmental temperatures other than 20 °C become correspondingly greater. In recognition of the possible consequences of mean environmental temperatures other than 20 °C, it is not uncommon to find the following procedures in use:

(a) Special gauging or working standards made of nominally the same material as the workpieces;

(b) Computation of corrections which are applied to the indicated values of length.

As the working tolerance decreases, both of these procedures fail to be satisfactory because of the magnitude of the uncertainty of nominal differential thermal expansion (see Section E2) and the length uncertainty due to temperature measurement (see Section 3.3.6).

E1.2 Measurements other than length. Procedures and formulae for the assessment of the effects of mean environmental temperatures other than 20 °C as simple and straightforward as those presented in this technical report are not usually possible in cases other than length measurements.

For example, consider the case of an iron bedway casting of a machine. Because the casting may have both thick- and thin-walled sections, the physical composition of the material may not be homogeneous, resulting in a non-uniform coefficient of thermal expansion. The magnitude of such a variation in thermal expansion coefficient may be as much as 5%. If the non-uniformity is distributed as a vertical gradient, raising or lowering the mean temperature will result in a bending like that produced by a vertical temperature gradient. This effect is the same as that observed in the well-known bi-metal strip and can be called a "bi-metal effect."

The bi-metal effect in structures of nominally one material is relatively small compared with the effect of temperature gradients. For example, a base casting like that mentioned above would have to be subjected to a temperature offset of 10 °C before the bending approaches that induced by an upper and lower surface temperature difference of only approximately 0,5 °C. However, in structures composed of two or more greatly dissimilar materials that are assembled at 20 °C, the bi-metal effect can be quite significant. In such cases the effect of mean temperature other than 20 °C can be properly estimated only by taking into account the thermal stresses that exist.

Evaluation of the effects of mean temperatures other than 20 °C requires that the net effect of the distortions of both working standard and workpiece be determined.

E2 Consequences of uncertainties of coefficients of thermal expansion and temperature

There are two kinds of uncertainties that arise when the effects of mean temperatures other than 20 °C are computed. They are the uncertainties in the values of the temperatures and in the coefficients of thermal expansion that are used in the computations.

Values of temperatures used in computations can be in error because of errors in measurements, defects in the instruments used in making the measurements or because of the location at which the measurement is made. For example, the thermometer used may be inaccurately calibrated or have a built-in source of error such as the self-heating effect found in resistance thermometers. Because of the self-heating effect, resistance thermometers can be very precisely calibrated in liquid baths and give erroneous readings on metal surfaces or in air because the heat transfer process is quite different in these cases.

Location of the temperature-measuring sensor is important because of possible gradients. Use of room air temperature values may introduce errors of 1 °C or more. Readings of direct-contact probes are more reliable but are still subject to error because of gradients within the object whose temperature is being measured or improper mounting of thermometers to the object. An effective means of assessing the validity of a given location is to compare temperatures at several locations.

The standard procedure for estimating the effects of uncertainty of nominal differential thermal expansion is to require that workpiece and working standard temperatures be measured to determine worst-case deviations from 20 °C. This procedure takes into account the effects of gradients in the apparatus, as well as in the room in which it is located.

If workpiece and working standard temperatures are not measured, the computations must include consideration of the larger uncertainties in the temperatures used in computing the estimation of the effects of temperatures other than 20 °C.

With proper attention to the simple, well-established rules of precision thermometry, uncertainties due to temperature measurement can be reduced. There are, however, many questions regarding the proper mounting of thermometers, and in actual measurement situations in the factory, temperature measurement errors can be large. The effects of uncertainties of coefficient of thermal expansion values are also difficult to overcome.

Coefficient of thermal expansion data are published in tables in many handbooks and other sources. These values cannot be used without consideration of uncertainties which arise because:

(a) The material of the elements of the measurement system--workpiece or working standard or both--differ from the material for which the data are given. The differences may be in chemical composition, physical composition, or both.

(b) The published values are usually the result of averaging data from several experiments and from several experimenters. Consequently, the data reflect the effect of experimental bias.

(c) The published values are valid only for temperatures other than 20 °C or for a range of temperature other than that of the computation.

The National Institute of Standards and Technology (formerly the National Bureau of Standards), in calibrating steel gage blocks, assumes an uncertainty of the coefficients of thermal expansion of ±5% when the heat and mechanical treatment of the steel is known. The variation of the coefficient is (1) about ±3% among many heat treatments of steel of nominally the same chemical content, (2) about ±10% among several heat treatments of the same steel, and (3) about ±2 % among samples cut from different locations in a large workpiece of steel that has been fully annealed. Hot or cold rolling will cause a difference of about ±5 %. Values closer to ±20 % are used in the ISO *Guide to the Expression of Uncertainty in Measurement* [ISO TAG 4:1995].

Other materials have uncertainty in the coefficient of thermal expansion due to the effects of chemical contamination or physical structure. Some materials have grain structure effects which cause thermal expansion coefficients to vary with direction.

The typical thermal expansion measurement is conducted with an apparatus called a dilatometer in which a specimen, usually rod shaped, is heated and its change of length measured. Another form of dilatometer measures change of volume by liquid displacement, resulting in a coefficient of volumetric thermal expansion. For isotropic materials, the coefficient of volumetric thermal expansion has a value approximately three times that of the coefficient of thermal expansion.

The fact that the typical test specimen bears little resemblance to real workpieces, with consequent uncertainties in composition and treatment not reflected in experimental data scatter, suggests that uncertainties can be reduced by direct measurement of each specific object, or full-scale dilatometry.

Figures E.1 and E.2 represent two common ways in which thermal expansion data are presented in the literature. Figure E.1 is a synthetic case deliberately oversimplified for the purposes of this discussion. Figure E.2 is an actual case.[1] Note that Figure E.1 is a plot of change of length, ΔL, as a function of temperature, where ΔL is defined as zero when the temperature is 20 °C. This is the usual form of raw data from dilatomer experiments.

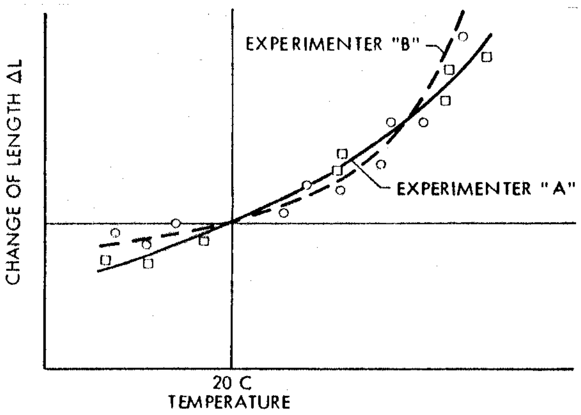

Figure E.1: Synthetic experimental results of thermal expansion measurements.

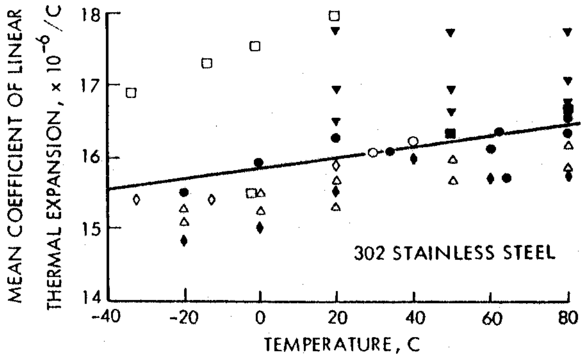

Figure E.2: Experimental data on the thermal expansion coefficient.

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[1] Data courtesy of Richard K. Kirby.

Figure E.2, on the other hand, is a plot of the mean (or average) coefficient of thermal expansion from 20 °C [see Equation (1), Clause 3.1],

$$\alpha(20,T) = \frac{L_T - L_{20}}{L_{20}(T-20)} \tag{E.4}$$

plotted at T. The data for $T = 20$ °C are derived from the slope of the thermal expansion, $d\,\Delta L/dT$, at that special temperature.

Figure E.2 gives results from several investigators. Figure E.1 shows how two investigators may obtain differing results that are reflected in Figure E.2. Both figures show (1) the scatter of experimental data and (2) the nonlinear nature of thermal expansion relative to temperature. Data of this type are the source of all tabulated coefficient of thermal expansion data. The published value, however, varies according to how the experimental data are interpreted. For a single investigation, the value depends on how the trend is interpreted, i.e., how the average curve is fitted. For multiple investigations, the value depends on how the data are averaged. In any case, these variations lead to increased uncertainties due to thermal effects (see Figure E.3).

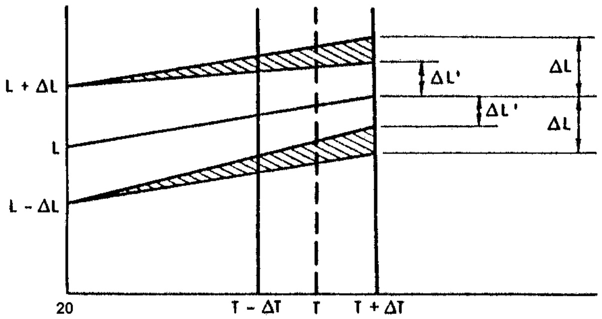


Figure E.3: Effects of uncertainties of coefficients of expansion on length measurement uncertainties.

For example, published values for pure or elemental aluminium are reported as 23.6 x 10^{-6}/°C at 20 °C in Metals Handbook and 22.4 x 10^{-6}/°C at 20 °C in Machinery's Handbook. Also, in Metals Reference Book, Table 2, the average from 20 °C to 100 °C is given as 23.9 x 10^{-6}/°C; and in Metals Reference Book, Table 1, the average from 0 °C to 100 °C is given as 23.5 x 10^{-6}/°C.

E3 Estimation of the Consequences of Environmental Temperature Variation

A good estimation of the consequences of temperature variation can very seldom be obtained by direct calculation. Therefore, the procedures described in this section are based on an experimental approach to the estimation.

The basic experimental procedure used to measure the effect of temperature variation is the drift test which is described in E3.1. Drift test results can be interpreted in a variety of ways to obtain an estimation of Environmental Temperature Variation Error. One method is described more fully in E3.2, along with other methods of interpreting drift test results that are not standard but may be useful because they are less conservative and may provide guidelines for negotiating the acceptability of thermal effects, bias, and uncertainty in special cases.

The rationale for both the drift test and the estimation of environmental temperature variation error is given in E3.3 in an explanation of the concept of the 3-element system.

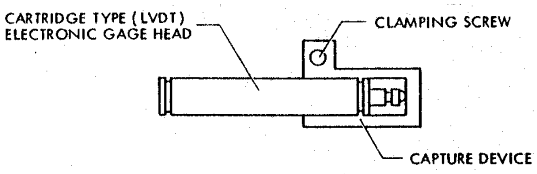
E3.1 Drift test procedure

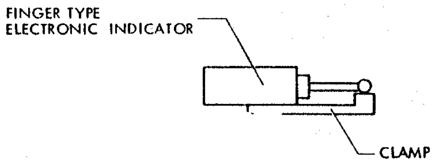
E**3.1.1 Equipment.** The object of a drift test is to record relative displacement in a 2-element system (see Clause E3.3). The most direct method utilizes electronic indicators whose output is recorded by a computer. Some measurement processes, such as the measurement of flatness with an optical flat and monochromatic light or an indicating micrometer, do not lend themselves to the use of automatic recording. Therefore, in some cases it will be necessary for a human operator to observe the drift and record numerical values and corresponding times. These data can be subsequently hand-plotted.

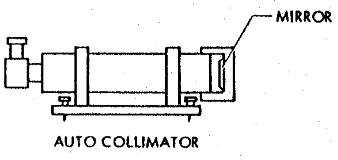
It is strongly urged, however, that, wherever possible, sensitive electronic indicators and automatic data-acquisition equipment be used.

Though a drift test can be performed without any necessity for knowledge of temperature variation, it is often advisable to record one or more temperatures either for use in later correlation of two drift tests or for reference if temperature variation is to be later accepted as a method of monitoring the process for validation of the environmental temperature variation error estimate.

Just as in the case of displacement measurements, it is strongly urged that all temperatures be automatically recorded. For this purpose, recording resistance element thermometers, especially those with thermistor sensors, are recommended.







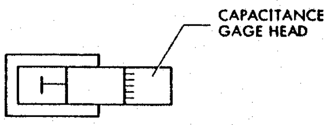


Figure E.4: Schematics for conducting "blocking" tests on transducers.

E3.1.2. Equipment testing

E3.1.2.1. Displacement transducers. Aside from the usual calibration checks, electronic indicators should be checked for possible sensitivity to the thermal environment in which the drift test is to be performed. An "electronics drift check" should be conducted by blocking the transducer and recording the output for at least the same period of time as that of the drift check to be performed. "Blocking" a transducer is to make it effectively indicate on its own frame, base, or cartridge. Figure C.4 shows, among others, a cartridge-type linear variable differential transformer blocked by means of a cap or capture device which holds the indicator armature in a fixed position relative to the cartridge.

During the electronics drift check, the entire displacement recording system should be located as nearly as possible to where it will be during the drift test.

Electronics drift tests have been useful in proving that, in many cases where electronic indicators have been the suspected source of drift, they were innocent and the real cause was thermal drift. The commercially available cartridge-type LVDT gage heads have been proven many times to be especially free from drift.

E3.1.2.2. Temperature recording systems. The temperature-measuring and recording apparatus should be calibrated and characterized for response and drift.

For many practical situations, resolution of 0,1 °C is adequate. This resolution and the time constants of sensing elements for air temperature sensors should be chosen to suit the measurement task and environmental conditions. Air probes must be shielded from possible radiation effects.

E3.1.2.3. Preparation of system for test. An essential feature of the drift test is that conditions during the test should approximate the normal conditions for the process. Therefore, before the test is started, normal conditions must be determined. The step-by-step procedure followed in the subject process must be followed in the same sequence and with the same timing as in the drift test. This is especially important in terms of the actions of human operators in mastering and all preliminary setup steps. With as little deviation from normal procedure as possible, the displacement transducers should be introduced between the workpiece (or working standard, depending on the type of drift check) and the rest of the measurement loop such that it measures relative displacement along the line of action of the subject measurement process.

The temperature sensor should be placed so as to measure a temperature which is correlatable with the drift. Some trial and error may be necessary. In the extreme case, temperature sensors may have to be placed to measure the temperatures of all of the active elements of the measurement loop.

E3.1.2.4. Representative time period for a drift test. Once set up, the drift test should be allowed to continue as long as possible, using normal operating conditions. In situations where a set pattern of activity is observed, its duration should be over some period of time during which most events are repeated. For example, if the acceptance tests on a measuring instrument are to take a day, the drift test should be run for a day.

E3.1.2.5. Procedure. E_{TVE} is the range of the dimensional variation due to environmental temperature variation. After the drift test, the displacement transducers and the temperature recording apparatus should be readjusted.

E3.1.2.6. Example drift test results. Figures E.5 and E.6 are results from drift tests conducted on a measuring instrument. Figure E.5 is the drift recorded over a 24-hour period for a system consisting of the working standard and comparator. Figure E.6 is the drift recorded over the succeeding 24-hour period for a system consisting of the workpiece to be measured and the comparator. In both cases, ambient temperature at a point near the gage was recorded and is plotted in the corresponding figures.

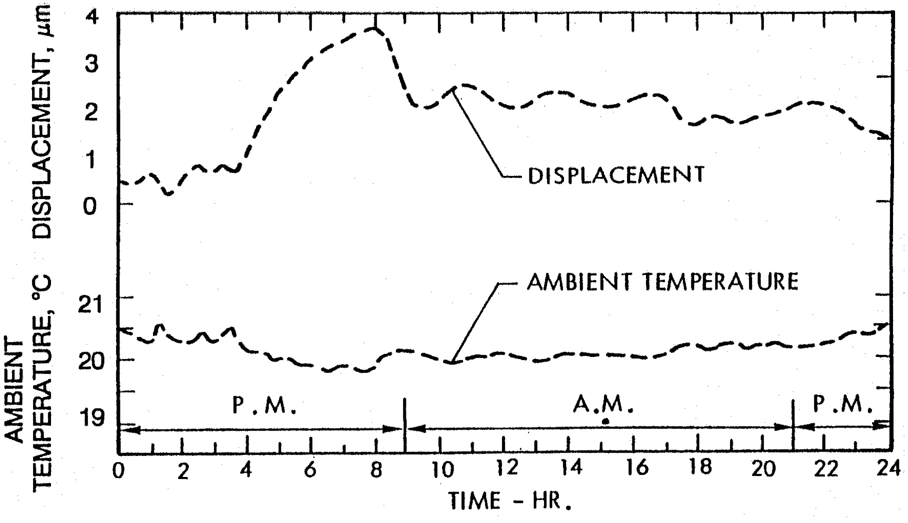


Figure E.5: Drift of a working standard and comparator system.

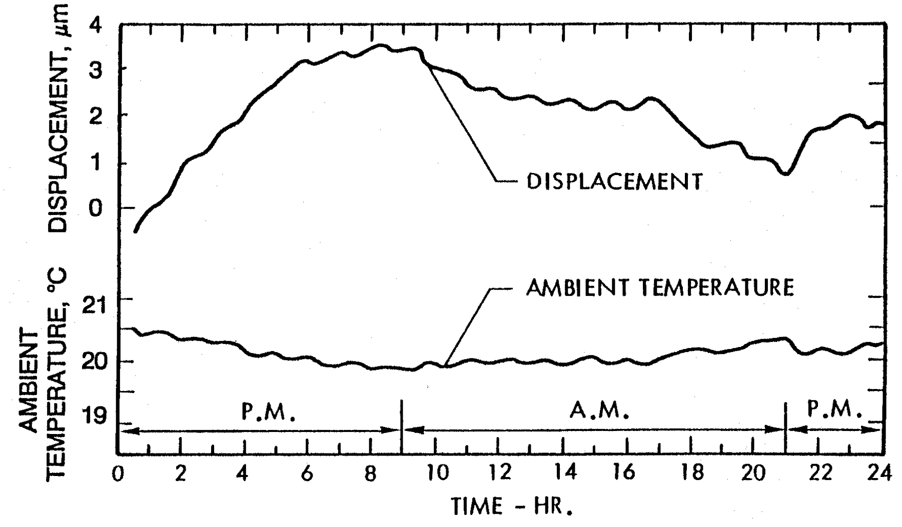


Figure E.6: Drift of a workpiece and comparator system.

E**3.1.2.7. Other drift tests.** For specific instruments or machines, other standards recommend different types of drift checks. For example, see ANSI B89.1.12M, ANSI B5.54, and ISO 230-3 Draft.

E3.2 Environmental temperature variation

Figure E.7 shows the results of both workpiece/comparator and working standard/comparator drift tests from Figures E.5 and E.6 superimposed on each other. In this case, ambient temperature readings were obtained simultaneously with each drift test for the purpose of approximating the proper phase relationship. The two sets of data were superimposed according to the time of day, which appears to give a good overall agreement in ambient temperature variation.

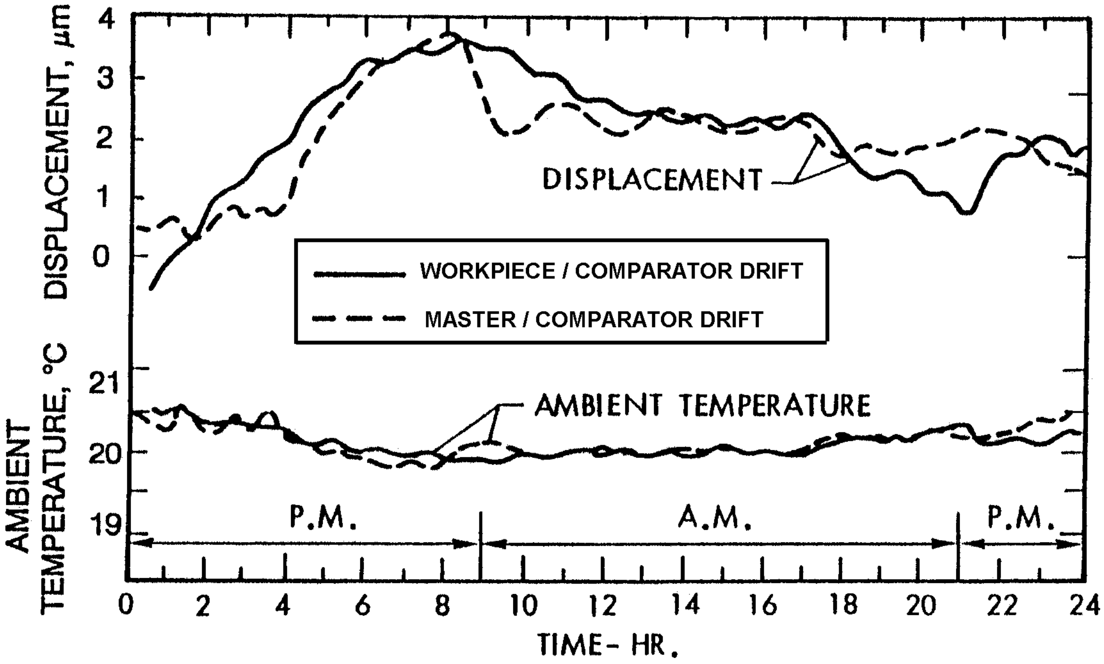


Figure E.7: Results of workpiece/comparator and working standard/comparator drift tests of a gage in an Inspection Shop. Tests were conducted on successive days. The data are superimposed on clock time.

The ambient temperature variation on the two successive days has a well-defined 24-hour component with an amplitude of about 0,8 °C. Superimposed on this are higher frequency components with periods of from ½ to 1½ hours. From these data it is possible to compare the 24-hour cycle characteristics because of the repeatability of the environment at this frequency; but phase relationships at the higher frequencies are not easily discernible.

At the 24-hour frequency, the working standard/comparator and workpiece/comparator drift curves are in phase and have very nearly the same amplitude. This is a classic example that shows the importance of measuring cycle time because the larger amplitudes of drift are associated with the low frequency, whereas the smaller amplitudes of drift are associated with the higher frequencies.

For short measurement cycle times, say 1 hour, the procedure for evaluating environmental temperature variation given in Section E3.1 results in an E_{TVE} = 1.5 µm. For measurement cycle times of 12 hours or more the E_{TVE} = 3 µm.

When the quality of the drift data permits, it is sometimes possible to apply the more precise evaluation methods discussed in Section E3.3 which are less conservative. In the example of Figure E.7, little is gained by this procedure because the maximum difference between the two drift curves, which corresponds to the possible error for short measurement cycle times, is still about 1.5 µm. This is probably because of non-repeatable components of temperature variation in the two days testing. The day on which the working standard/comparator drift test was performed appears to have had more severe high frequency temperature components. This discrepancy appears to exaggerate the true workpiece/working standard relative drift. Further drift tests to obtain results for more consistent temperature variations would be advisable in this case if environmental temperature variation is the major thermal effect in this measurement process.

E3.3 The three-element system concept

The magnitudes of the effects of temperature variation are dependent on the structure of the measurement apparatus and not only on the size and composition of the workpiece and working standard as was true in the previous sections. Also unlike the other components of thermal error, environmental temperature variation depends on the work habits of the person making the measurements.

One of the simplest structures is that encountered in the measurement of the length of an object with a gage block and a column comparator. Figure E.8 shows a schematic representation of such a system. As can be seen, it consists of a workpiece, a working standard (the gage block) and a comparator. Thus, the system consists of three elements. Here, the system is called the comparator method; it may also be referred to as the substitution method.

In Figure E.8, each element is shown to have a characteristic length : L_w = workpiece length, L_s = working standard length, and L_i = comparator length. In the measurement process, L_i is first set equal to L_s, then L_w is checked to see if $L_w = L_i$.

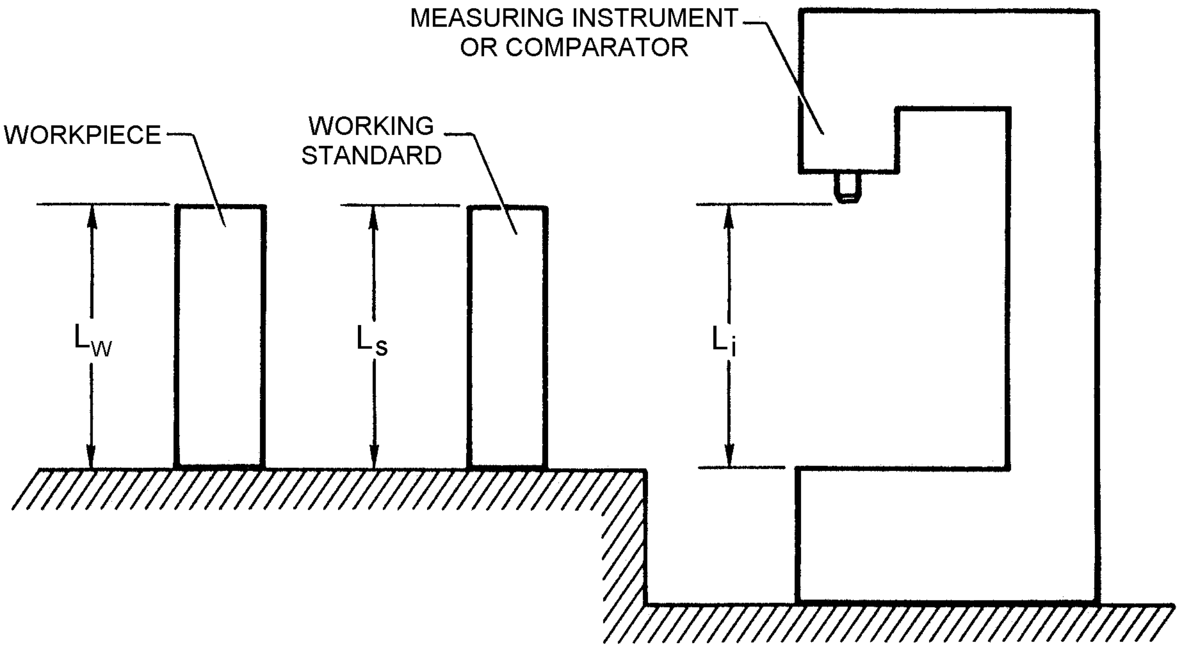


Figure E.8: The three elements of a length-measuring system.

If there were no temperature variations, the measurement process would be straightforward. However, because of temperature variations, heat is constantly being exchanged between the three elements and the changing environment.

If the time constants of all three elements are not the same, they may respond to temperature variations in such a way that all three elements will never simultaneously have the same temperature. Even if the time constants were all the same and their temperatures always equal, they may not have the same length, except when all are set at 20 °C, because of different coefficients of thermal expansion.

For each element, the time constant, length, and coefficient of thermal expansion determine its dimensional response to temperature variation.

Figure E.9 shows the dimensional response of the three elements of Figure E.8 for an assumed sinusoidal ambient temperature variation. For simplicity, the hypothetical system consists of three elements of the same material but different time constants, the largest being that of the working standard, the smallest being that of the workpiece with the time constant of the comparator between those of the other elements.

As can be seen, the three-dimensional thermal responses differ in amplitude and phase. It should be noted that dimensional response data in this form are rarely obtainable because they require the use of an independent apparatus that must itself be unaffected by temperature variation.

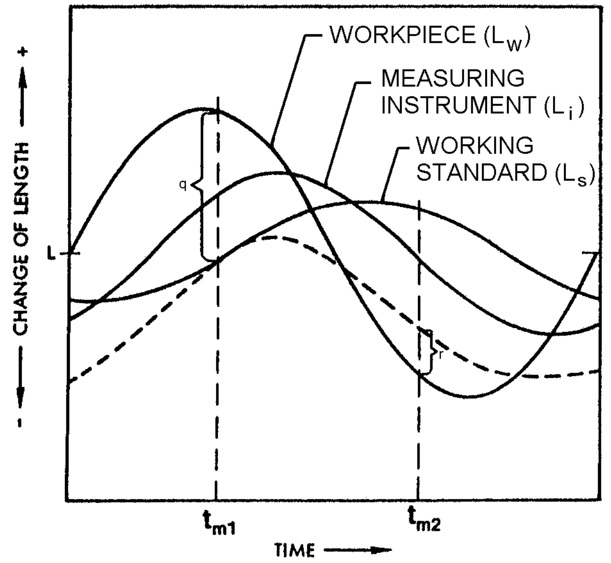


Figure E.9: Sample steady-state dimensional response of a 3-element system to a sinusoidal ambient-temperature variation.

The data of Figure E.9, if they were obtainable, can easily be interpreted for an estimate of the environmental temperature variation. It is only necessary to consider the effect of the measurement cycle as follows.

Suppose that at time t_{m1} the comparator is mastered. The act of making L_i and L_s equal causes the dimensional response curve of the comparator to be shifted parallel to itself (the comparator is "zero shifted") as shown by the dashed curve. If the workpiece is checked without delay after mastering, it is found to be too large by the amount q. If, instead, the workpiece is checked much later, say at time t_{m2}, the workpiece will be found to be too small by the amount r. If the comparator is remastered at time t_{m2}, the comparator curve is again shifted, resulting in new magnitudes of possible error.

Because environmental temperature variation causes a variation of the differences of the characteristic lengths, it is possible to separate the three-element system into two two-element subsystems. For example, Figure C.10 shows the two curves that result when the comparator variations (L_i) are subtracted from the workpiece and working standard dimensional responses (L_w-L_i and L_s-L_i). These data might have been obtained by recording the output of an electronic indicator, such as is found used on modern column comparators, when the workpiece and working standard are, successively, in the comparator with the indicator contacting the workpiece and working standard, respectively. Data such as these are obtained using drift tests as described in Section E3.1.

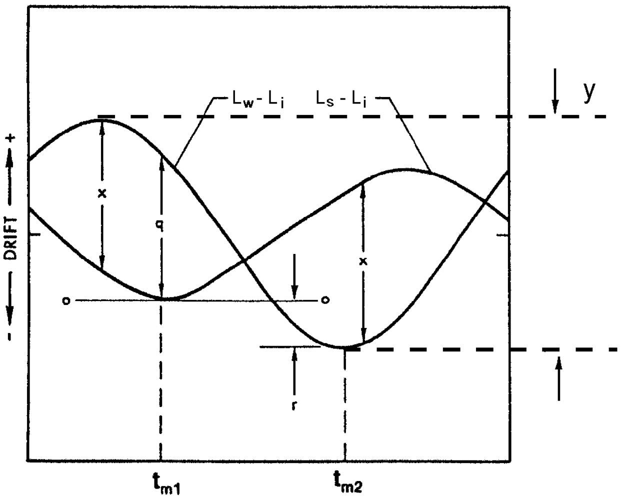


Figure E.10: Relative drift components for a three-element system. L_w-L_i is the relative drift of workpiece and comparator, and L_s-L_i is the relative drift of working standard and comparator.

The main problem in interpreting such data results from the fact that it is not possible to conduct simultaneous workpiece/comparator and working standard/comparator drift tests. Consequently, additional data are required to determine the proper phase relationship between the two recorded drift curves or the possible consequences of unknown phase relationship must be considered in the estimate of environmental temperature variation.

Because the data of Figure E.10 have been constructed from the data of Figure E.9, no phase uncertainty exists and the environmental temperature variation error can be extracted easily. For example, for a mastering cycle occurring between times t_{m1} and t_{m2} and a measurement of the workpiece without delay, the possible error, q, is simply the difference between the two curves.

The effect of mastering is to establish a new baseline for the workpiece/comparator drift curve (L_w-L_i). This new baseline is shown in Figure E.10 as the line (0-0). If the measurement of the workpiece takes place at time t_{m2}, the resultant error is r as previously shown.

If a series of similar workpieces are inspected between times t_{m1} and t_{m2}, the errors due to environmental temperature variation range from $+q$ to $-r$.

If the times at which mastering occurs are unknown and unpredictable and the measurement cycle time is very short (mastering with each measurement and negligible delay before the workpiece is inspected), the possible error is $\pm x$, or the maximum difference between the two drift curves at any given time. Because of the short measuring cycle time the comparator is slaved to the working standard so that the comparator contributes nothing to the error. The error, therefore, is $(L_w - L_i) - (L_s - L_i) = (L_w - L_s)$.

This error is dependent only on the difference between the working standard/comparator drift and the workpiece/comparator drift and the time at which the measurement is made. If the measurement cycle time is longer than the period of the temperature oscillation, the maximum possible error is $\pm y$, or the maximum difference between the two drift curves regardless of time. Note that y is slightly larger than x.

In length-measuring processes, however, a three-element system is always found. For example, consider the case shown in Figure E.11 of a measuring machine or machine tool with a leadscrew (or a linescale) serving as working standard. In Figure E.11, the measurement process is shown to consist of changing from position (a) to position (b). The analogy between this case and the simple three-element system of

Figure E.8 is seen if it is realized that in the two configurations; the comparator is composed of a portion of the lead-screw (or a linescale), the nut, and the table support for the workpiece. These elements, though appearing to change, remain in a structural loop, while the workpiece and working standard exchange places as members of the loop. Note that since the working standard and the workpiece are simultaneously in the comparator, only one drift check is needed.

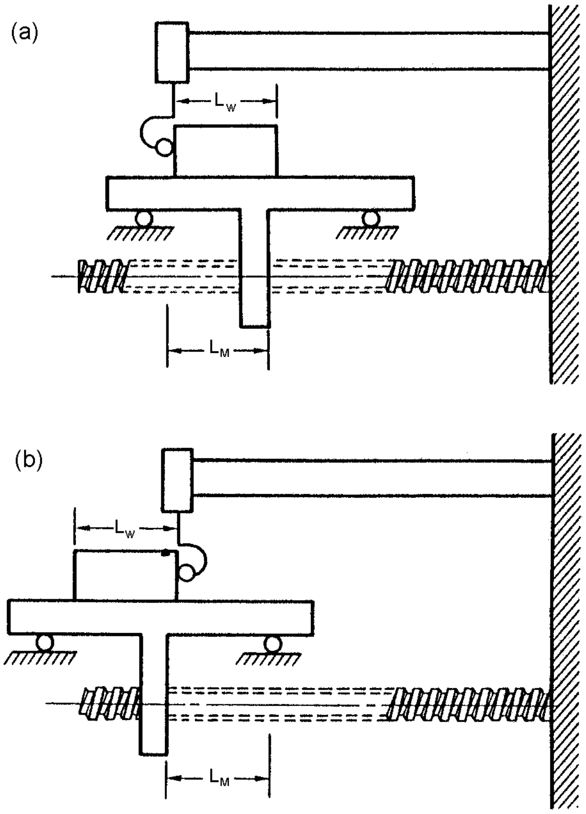


Figure E.11: Schematic of setup used to measure a workpiece on a gage with a lead-screw working standard. The measuring sequence is a change from (a) to (b). The indicator is highly idealized.

The case shown in Figure E.12 is that of a 25-mm indicating micrometer used as a comparator. The working standard is a gage block.

In Figure E.13 the same micrometer is brought to its null position and a zero correction is made before the workpiece is measured. In this case the working standard is that portion of the screw that is withdrawn to make room for the workpiece. The rest of the micrometer forms the comparator.

Consider now a 50 mm indicating micrometer and the following case. The workpiece is 35 mm in diameter. A 25 mm gage block is used to master the micrometer. The working standard in this case is the gage block plus that portion of the screw, approximately 10 mm long, which is withdrawn to make room for the workpiece (see Figure E.14).

These cases show how the working standard and comparator can be changed by changes in the operating procedure.

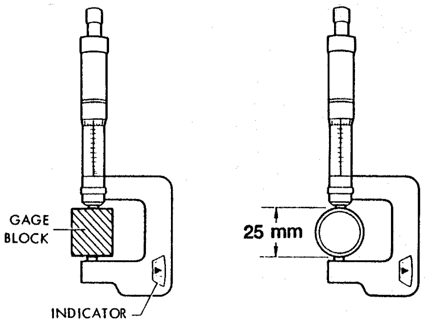


Figure E.12: Use of a micrometer as a comparator.

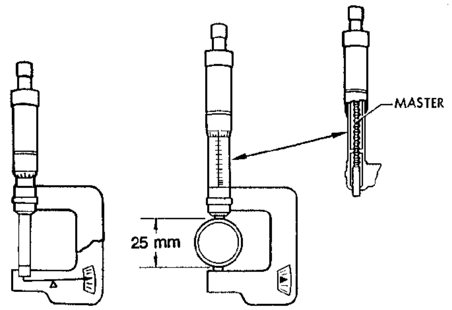


Figure E.13: A measurement made with a micrometer that has been zeroed. The working standard is now a portion of the screw.

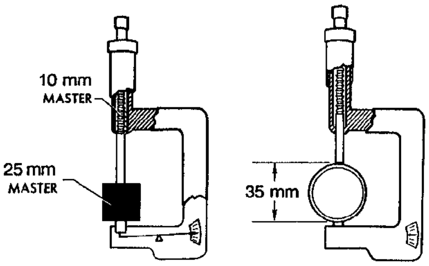


Figure E.14: A combination case where a portion of the screw and a portion of the comparator are used in the length measurement.

E4 Thermal error index

This technical report does not recommend values for the thermal error index. Such values cannot be stated without regard to other sources of error in the measurement process. For example, a thermal error index of 10% assigns to thermal effects that fraction of the tolerance that is usually considered to include the composite effects of all error sources. In any given case, the permissible values depend on the degree of control that is maintained over all aspects of the measurement process, including the skill level of personnel.

One way to reduce a thermal error index is to increase the working tolerance. Consequently, it serves as a feedback device to inform management and designers of the degree of difficulty posed by a specified tolerance. The thermal error index does nothing more than estimate the maximum possible uncertainty due to the thermal environment affecting a particular measurement process. It does not establish the true magnitude of error in any measurement. It serves to remove doubt about the existence of errors and to establish a system of rewards and penalties to processes that are combinations of techniques and conditions, some good and some bad.

A thermal error index evaluation penalizes a measurement process on four counts:

a) Existence of temperatures other than 20 °C

b) Existence of temperature variations

c) Existence of temperature gradients

d) Not applying the differential thermal expansion correction

The same evaluation rewards good practice by decreasing the thermal error index for (1) attempting a correction for temperatures other than 20 °C, (2) keeping environmental variations to a minimum, and (3) maintaining acceptable temperature gradients. The act of performing the evaluation results in the knowledge of what techniques or conditions can be changed to achieve the greatest improvement with the least effort. For example, if environmental temperature variation error is found to be the greatest source of error, the measurement cycle time may be reduced such that the thermal error index is reduced to an acceptable value. Thus, by more frequent mastering, at some nominal increase in operating expense, possible misapplication of capital to improve temperature control is avoided.

Bibliography for Annex E

ANSI/ASME B89.4.1, 1997, *"Methods for Performance Evaluation of Coordinate Measuring Machines."*

ANSI/ASME, *"Temperature and Humidity Environment for Dimensional Measurement,"* American National Standard B89.6.2-1973, American Society of Mechanical Engineers, New York, 1973 (R1979, R1988).

Bryan, J., *"International Status of Thermal Error Research,"* Annals of the CIRP 39/2, 1990.

Bryan, J., McClure, E. R., Pearson, J. W., Brewer, W., *"Thermal Effects in Dimensional Metrology",* ASME 65, Prod. 13, 1965.

Bryan, J. B., Donaldson, R. R., McClure, E., Clouser, R. R., *"A Practical Solution to the Thermal Stability Problem in Machine Tools,"* SME, MR-72-138, 1972.

Estler, W. T., *"High-accuracy displacement interferometry in air,"* Applied Optics 24, No. 6, March 1985.

ISO (International Standards Organization*), "Guide to the Expression of Uncertainty in Measurement,"* ISBN 92-67-10188-9, Geneva, Switzerland, 1993.

McClure, E. R., D. Eng. Thesis, *"Manufacturing Accuracy Through the Control of Thermal Effects,"* Lawrence Livermore National Laboratory, Livermore, CA, UCRL-50636, 1969.

Zhang, G., Veale, R., Charlton, T., Borchardt, B., Hocken, R., *"Error Compensation of Coordinate Measuring Machines,"* CIRP Annals, Vol. I, 1985.

Annex F

Example of the uncertainty in length measurement due to thermal effects

F1 Example problem: measurement of a workpiece using a comparator

The length of a workpiece of nominal length L_w is to be measured by comparing it with a known working standard of length L_s. The lengths are nominally the same even when the workpiece is being measured on a measuring machine and the known working standard is the machine scale. The comparison is performed by a comparator, using a short-range displacement indicator whose change in output between adjustment and measurement is d. In general, the working standard, the comparator, and the workpiece are not at 20 °C, and their temperatures are not the same, and are changing with time. They also have different coefficients of thermal expansion. It is desired to evaluate the standard uncertainty component in this measurement resulting from thermal errors alone.

The comparator measures the difference in lengths

$$d = L_w(1+\alpha_w\theta_w) - L_s(1+\alpha_s\theta_s)\,, \tag{F.1}$$

where

L_w is the measurand, that is, the length at 20 °C of the workpiece being measured;

L_s is the length of the working standard at 20 °C as given on its calibration certificate;

α_w and α_s are the coefficients of thermal expansion of the workpiece and the working standard;

θ_w and θ_s are the deviations in temperature from 20 °C, respectively, of the workpiece and the working standard.

F2 The mathematical model

From Equation (F.1), the measurand is given by:

$$L_w = [L_s(1+\alpha_s\theta_s) + d]/(1+\alpha_w\theta_w)$$

or

$$L_w \approx L_s + d + L_s(\alpha_s\theta_s - \alpha_w\theta_w) \tag{F.2}$$

assuming $\alpha\theta$ is small. The third term on the right-hand side of this expression is the differential thermal expansion between the working standard and the workpiece. In the treatment that follows it is assumed that this differential expansion has been accounted for in the calculation of the length of the workpiece.

F3 Uncertainty evaluation

Assuming that the quantities α_w, α_s θ_w, and θ_s are uncorrelated, and applying the law of propagation of uncertainty to Equation (D.2) yields (setting $L_W=L$) :

$$u_c^2(L) = [1+(\alpha_s\theta_s - \alpha_w\theta_w)]u^2(L_s) + u^2(d) + u_{DE}^2(L) + u_{TM}^2(L), \tag{F.3}$$

where

$$u_{DE}^2(L) = L_s^2\theta_s^2u^2(\alpha_s) + L_w^2\theta_w^2u^2(\alpha_w), \tag{F.4}$$

is the component due to uncertainties in the coefficients of thermal expansion (see Clause 3.2.5), and

$$u_{TM}^2(L) = L_s^2\alpha_s^2u^2(\theta_s) + L_w^2\alpha_w^2u^2(\theta_w), \tag{F.5}$$

is the component due to uncertainties in the temperatures (see Clause 3.3.6).

As a numerical example, consider a steel workpiece (α_W = 12 × 10^{-6}/°C) of nominal length 500 mm measured by comparison with a glass scale (working standard, α_S = 8 × 10^{-6}/°C). The environment is assumed to be varying about a mean temperature of 25° C. A one-hour drift test has been conducted, revealing a peak-to-valley range of length variation of 12 µm. The temperatures of the workpiece and the glass scale are measured using calibrated thermistors; the mean temperature of the workpiece is 26 °C (θ_W = 6 °C) and the mean temperature of the scale is 24° C (θ_S=4 °C). The workpiece tolerance is 50 µm (a typical production tolerance), and the adjustment cycle is assumed to be one hour.

F3.1 Uncertainty of the calibration of the length of the working standard, $u(L_s)$. If a working standard in the classical sense were used for comparison purposes, this uncertainty would be taken from the calibration certificate as described in the ISO *Guide*. If the measurement were performed on a measuring machine it would be appropriate to use the results of linear displacement accuracy tests of the axis used for the measurement, assume a uniform (rectangular) distribution with the appropriate limits, and compute the standard uncertainty. In any case, this is not a dimensional uncertainty caused by thermal effects and in the following it is assumed that $u(L_s)$ = 0.

F3.2 Uncertainty of the measured difference in lengths, $u(d)$. In the ideal situation, multiple measurements would be made on the workpiece with these measurements spanning a period of time. The distribution of these measurements would be examined and, depending upon the distribution, the standard uncertainty could be calculated according to the guidelines. In the present situation where the workpiece, the working standard, and the comparator are at different temperatures, with different time constants, and the environmental temperature one is changing, the values obtained for d would have a wide range. The complexity of this situation is discussed in Annex E.

In the absence of such a definitive series of measurements one can only estimate this uncertainty, which will have several components. One of these components, called $u_{ETVE}(L)$, would be attributed to the changing temperatures of workpiece, working standard, and comparator with their different time constants and environmental couplings.

As a general procedure, it is recommended that this component be assessed by performing drift tests over a period of time comparable to the duration of the measurement. These drift tests, one for the working standard and one for the workpiece, will result in a range of values for d which is identical with the quantity E_{TVE} described in Annex E. Assuming that a uniform distribution of width E_{TVE} characterizes one's knowledge of the effects of environmental temperature variation, then:

$$u(d) = u_{ETVE}(L) = E_{TVE} / 2\sqrt{3} \,. \tag{F.6}$$

In this example, then:

$$u_{ETVE}(L) \approx 3.5\ \mu m$$

Note that this assumes that the measurement process takes one hour, the same as the duration of the drift test.

F3.3 Uncertainties of the thermal expansion coefficients, $u(\alpha_w)$ and $u(\alpha_s)$. The coefficients of thermal expansion have considerable uncertainty (see Section C.2). In its example in Annex H, the ISO *Guide* suggests a uniform distribution to characterize knowledge of α_w and α_s, with a range of $\pm 2 \times 10^{-6}\ °C^{-1}$, a suggestion followed in this example. The contribution to the measurement uncertainty from this source follows from

$$u_{DE}(L) = \sqrt{L_s^2\theta_s^2 u^2(\alpha_s) + L_w^2\theta_w^2 u^2(\alpha_w)} \tag{F.7}$$

where $u^2(\alpha_s) = u^2(\alpha_w) = [(4 \times 10^{-6})^2/12]\ °C^{-2}$

For this example, taking $L_s \approx L_w = L = 500$ mm, it follows that:

$$u_{DE}(L) = Lu(\alpha)\sqrt{\theta_s^2 + \theta_w^2}$$

so that

$$u_{DE}(L) \approx 4.2 \mu m \,.$$

F3.4 Uncertainties in the temperatures, $u(\theta_w)$ and $u(\theta_s)$. The measurements of the temperatures of the workpiece and the working standard also have uncertainties arising from thermometer calibration, sensor mounting, and electronic noise. Since the thermal expansion coefficients of workpiece and working standard are different, the uncertainty component from this source is

$$u_{TM}(L) = \sqrt{L^2\alpha_s^2 u^2(\theta_s) + L^2\alpha_w^2 u^2(\theta_w)} \tag{F.8}$$

or

$$u_{TM}(L) = Lu(\theta)\sqrt{\alpha_s^2 + \alpha_w^2} \tag{F.9}$$

where it is assumed that $u^2(\theta_s) = u^2(\theta_w) = u^2(\theta)$. The terms $u^2(\theta_w)$ and $u^2(\theta_s)$ must be evaluated by the person performing the measurements. In absence of other guidelines, it is suggested that one should assume that these errors are uniformly distributed with a range of ±1 °C for thermocouples and ±0,5 °C for properly calibrated thermistors.[2] For this example this uncertainty becomes:

$$u_{TM}^2(L)=(0{,}5m)^2(0{,}5°C)^2[(8\times10^{-6}°C^{-1})^2+(12\times10^{-6}°C^{-1})^2]/3$$

$$u_{TM}^2(L)\approx 4.3\,\mu\text{m}^2$$

$$u_{TM}(L)\approx 2.1\,\mu\text{m}$$

F**3.5 Uncertainty in the length measurement due to thermal effects.** Combining the terms from the previous expressions [Eqs. (D6), (D7), and (D8)] yields a standard uncertainty component for the measurement of length, $u_{cT}(L)$, of:

$$u_{cT}(L)=\sqrt{u_{ETVE}^2(L)+u_{DE}^2(L)+u_{TM}^2(L)} \tag{F.10}$$

The first term in the square root is the contribution due to environmental temperature variation, the second term is the contribution due to uncertainty in the thermal expansion coefficients, and the third term is the contribution due to uncertainty in the temperature measurement. For this example, the combination yields:

$$u_{cT}(L)\approx\sqrt{(12+17.3+4.3)\,\mu\text{m}^2}$$

or

$$u_{cT}(L)\approx 5.8\,\mu\text{m}.$$

None of the uncertainties are dominant.

F**3.6 Thermal error index (*TEI*).** If the user should choose not to perform the correction for nominal differential thermal expansion, then if there is a tolerance on the workpiece dimension, a *TEI* calculation must be performed. In this example, a steel workpiece (α_w = 12 ppm per °C) of length 500 millimetres is compared with a glass scale (α_s = 8 ppm per °C). The environment is at 25 °C and changing, the glass scale at 24 °C, and the workpiece at 26 °C. The tolerance is 50 µm. This case yields a nominal differential thermal expansion (Δ_{nDE}) of 20 µm. The *TEI* is given by:

$$TEI=\frac{2|\Delta_{nDE}|+4u_{cT}(L)}{TOL}\times100\%. \tag{F.11}$$

For this example the *TEI* is 126%. In such a case, it would be impossible to prove that the workpiece was acceptable, since the range of reasonably probable errors exceeds the width of the tolerance zone.

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[2] Standards laboratories, of course, can measure temperatures much more accurately than this, but this is a reasonable number for industrial measurements.

Bibliography for Annex F

ISO (International Standards Organization) /TAG4:1995*, "Guide to the Expression of Uncertainty in Measurement (GUM),"* ISBN 92-67-10188-9, Geneva, Switzerland.

ANSI/ASME, *"Temperature and Humidity Environment for Dimensional Measurement,"* American National Standard B89.6.2-1973, American Society of Mechanical Engineers, New York, 1973 (R1979, R1988).

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