Example of hypothesis testing, and of confidence intervals.

We have the following data measurement data from a process:

50.8028	48.8705	48.4132	51.1830	46.6311
49.6338	51.1584	53.1800	50.9440	49.8908

Questions: 1) Test the hypothesis that the true mean of this process is equal to 48.85 at the $\alpha = 0.05$ level.
2) What is the P-value for this hypothesis test ?
3) Test the hypothesis that the mean exceeds 49.0 at the $\alpha = 0.05$ level.
4) Find the 90% Two-sided confidence interval for the mean.
5) Find the 95% upper confidence interval for the variance.

Solution: First, find the sample mean, sample variance, and sample standard deviation.

$\bar{x} = 50.0708, \quad S^2 = 3.2920, \quad S = 1.8144$

1) now, develop the hypotheses. The null hypothesis will always be an equality,

so H_0: $\mu = 48.85$ (note the μ_0 is 48.85)
and H_1: $\mu \neq 48.85$

Because we don't know the true standard deviation, σ, we will use the t-test for this hypothesis.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{50.0708 - 48.85}{1.8144/\sqrt{10}} = 2.1277$$

The criterion for rejecting H_0 is that $|t_0| > t_{\alpha/2, n-1}$

$$t_{\alpha/2,\, n} = t_{0.025,\, 9} = 2.262$$

t_0 is not > 2.262, so we ***can not*** reject the null hypothesis at the $\alpha = 0.05$ level.

2) The P-value for a hypothesis test is the smallest level of significance (α) for which we would reject the null hypothesis. For our test, this means finding the α such that $t_{\alpha/2,\, n-1}$ is exactly equal to the test statistic t_0.

Looking in the table, we see that for 9 DOF, $t_{0.025,\, 9} = 2.262$ and $t_{0.05,\, 9} = 1.833$. Linear iterpolation gives us $t_{0.0328,\, 9}$ is about 2.1277.

The P-value would be equal to 2 x 0.0328 = 0.0656 because 0.0328 was the $\alpha/2$ value.

3) again, develop the hypotheses. The null hypothesis will always be an equality,

so H_0: $\mu = 49.0$ (note the μ_0 is 49.0)
and H_1: $\mu > 49.0$

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{50.0708 - 49.0}{1.8144/\sqrt{10}} = 1.8363$$

The criterion for rejecting H_0 is that $t_0 > t_{\alpha, n-1}$

$t_{\alpha, n-1} = t_{0.05, 9} = 1.833$

So, we meet the criterion for rejecting H_0 and conclude that the mean ***does*** exceed 49.0.

4) A confidence interval for a certain α can be defined as follows:

$$P(L < x < U) = 1-\alpha$$

$\alpha = 0.10$ will give use the 90% Two-sided confidence interval between L and U.

From our tables (or the book): $\bar{x} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$

So $$50.0708 - t_{0.05,9}\frac{1.8144}{\sqrt{10}} \le \mu \le 50.0708 + t_{0.05,9}\frac{1.8144}{\sqrt{10}}$$

and $$49.0191 \le \mu \le 51.1225$$

5) The upper confidence interval is defined as $P(x < U) = 1-\alpha$.

From the book: 2-sided: $\frac{(n-1)S^2}{X^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{X^2_{1-\alpha/2,n-1}}$

Upper: $\sigma^2 \le \frac{(n-1)S^2}{X^2_{1-\alpha,n-1}}$

So $$\sigma^2 \le \frac{(9)3.292}{X^2_{.95,9}} = \frac{(9)3.292}{3.33}$$

and $\sigma^2 \le 8.8973$