Lecture 4 – Projectile Motion

Administrative:

- Reading for Next Time: Sections 2/3 and 2/4 and Sample Problems 2/5 and 2/6.
- Chatroom Help Tomorrow 9 PM, WEBCT Chatroom 1.
- We will use WEBCT a lot in this class. You can download the notes before class if desired.
  - My experience is that people either say that if they have the notes during class they fall asleep because they know they already have the notes OR they say that if they have the notes it gives them time to pay attention in class and so they learn more. You can decide what works best for you.
  - Whatever works best for you, PRINT MULTIPLE PAGES OF NOTES PER PRINTED PAGE OR YOU WILL RUN OUT OF PRINT QUOTA! YOU CAN CHOOSE THIS OPTION WHEN YOU HIT PRINT COMMAND.
- Class Improvement Committee – need three volunteers.

Closure: Motion along a Linear Path

Highlights

- Particle – a physics/math name for an object that has dimensions that are small compared to the radius of curvature of its motions. In most cases an object can be considered to be a particle if when you imagine looking at the motions you care about, it is difficult to distinguish the shape of the object…like a shooting star, or an airplane in the sky, or a car when you look down form an airplane at 1000 feet.
- Kinematics – The geometrically characteristics of motion…if you are told how something will accelerate but you are not able to predict why it is accelerating in that way, you can still predict where it will be at different times. This works in 1-D (line) 2-D (plane) and 3-D.
- Rectilinear motion – Motion along a line. The simplest way something can move is when it is confined to a linear path…this of a car on a straight road: the driver may push the accelerator and the break in unpredictable ways, but the car stays on the road … hopefully.
- Position: Where something is located relative to some point you choose.
- Velocity: The rate of change of position. Mathematically we use differentially calculus.

$$\boxed{v = \frac{dx}{dt}}$$

- Acceleration: The rate of change of velocity.

$$\boxed{a = \frac{dv}{dt}}$$

Introduction to Motion on a Curved Path

Now we look at a more complex form of motion, the motion of a particle along a curved path in a plane. This brings out the very important idea of the time derivative of a vector quantity. It is important that you master this concept, as it will come up again and again throughout the class.

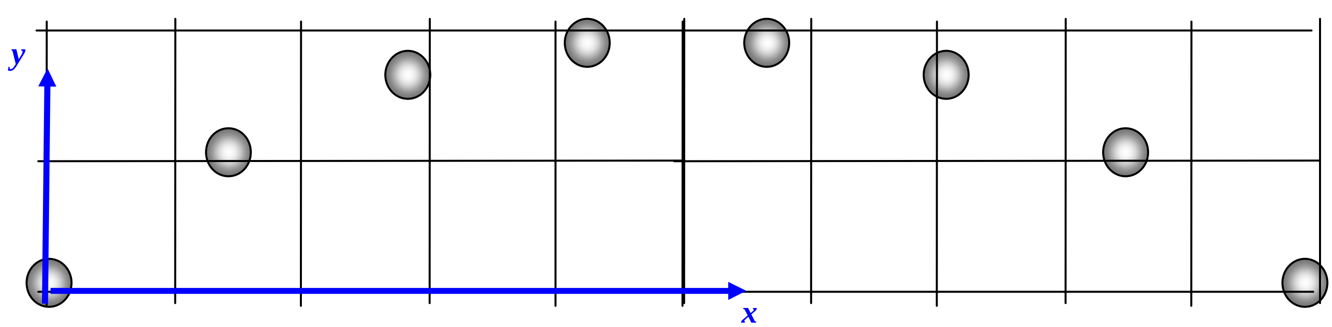
In addition, we are going to begin our discussion by talking about *vectors* in a general sense without regard to a *coordinate system* (e.g. rectangular, polar or spherical). <u>We can do this because vectors are mathematical objects that exist and can be manipulated (e.g. dot and cross products, derivatives and integrals etc.) without regard to a coordinate system.</u> It is only when we want to resolve the vectors into a coordinate frame that has physical meaning in a particular problem that we speak of the *representation* of a vector in that coordinate system. This is an advanced concept and I don't expect full understanding, but if you go on to more advanced classes, it will come up again.

Consider the motion of a ball. If I drop it straight down next to a tape measure, it accelerates at a constant rate and moves along a line. Now suppose we give it a velocity perpendicular to the ground. Then it moves in a plane. How do we describe this type of motion?

<u>Projectile Motion:</u>

One of the simplest forms of motion of this type is called projectile motion. It is called projectile motion because it what projectiles and somebody needed to give it an impressive sounding name. What is means is the motion of objects (particles) that are launched with an initial velocity that is not just straight up and down…that would be rectilinear motion…another fancy name for something you already know about. The ever-present influenced on these objects is gravity even though is necessary to consider air resistance when you have something that has a low density or in the case of an object with propulsion like a rocket, you might have to consider how the propulsion affects the motion.

So what happens when an object moves under the influence of gravity alone like the case when I throw a ball?

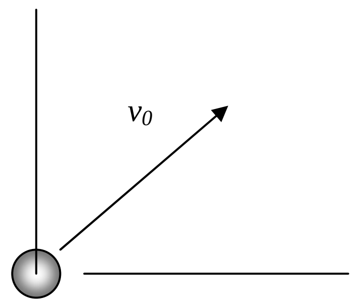


Qualitative Description – This Means no Numbers?

- What can you say about this motion?
- What are the *x* and *y*? What does it mean when I write *x(t)* and *y(t)*?
- What do I need to know to figure out how the ball moves?
- Do you know any words that describe the motion?
- Think of some everyday examples where this type of motion occurs?

Quantitative Description – this means with equations and numbers

- What piece of information do you need to know first?



$$\mathbf{v} = \mathbf{v_0}$$

- Why did I make this bold? I could also use the following information instead.

$$v = v_0 \text{ ; } Launch\ Angle = \theta$$

- What happens to the ball in the x-direction?

$$\ddot{x} = a_x = 0$$

- What happens in the y-direction?

$$\ddot{y} = a_y = -g$$

So for this case, the two equations we showed above are a particular set of differential equations that can be integrated to find the motion.

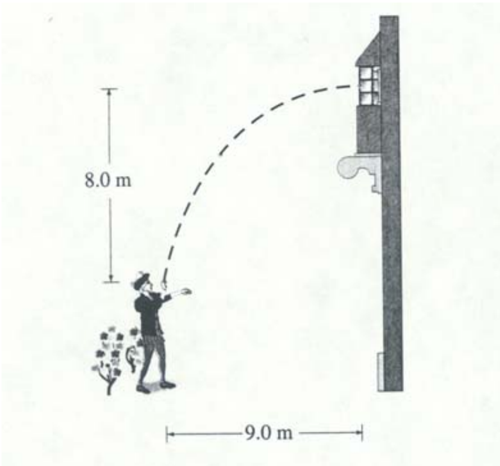
$$\ddot{x} = a_x = 0$$
$$\dot{x} = v_{0x}$$
$$x = v_{0x}t + x_0$$

$$\ddot{y} = a_y = -g$$
$$\dot{y} = -gt + v_{0y}$$
$$y = -g\frac{t^2}{2} + v_{0y}t + y_0$$

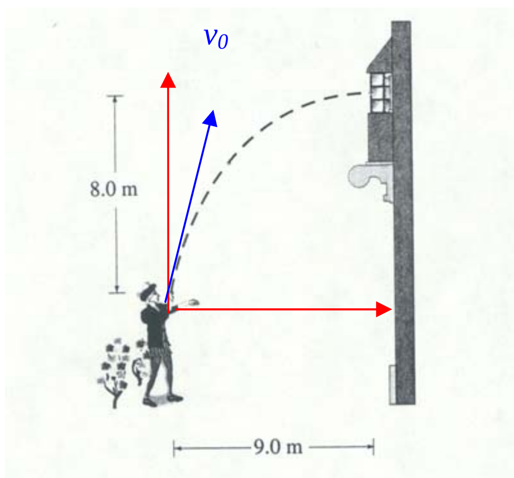
So this is what happens when you have a free flying object and you ignore air resistance. In these types of problems, you have to use all of this and look at what you know and what you don't know and then use the equations to solve for your unknowns.

Example: The infamous “Romeo Problem” from previous exam.

Problem 1 (40 points total): Romeo is throwing pebbles gently up to Julliet’s window, and he wants the pebbles to hit the window only with a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 meters below her window and 9.0 meters from the base of the wall. Calculate the initial vertical (v_{0y}) component of the velocity of the pebbles. Calculate the time (t_w) it takes the pebbles to reach the window. Calculate how fast the pebbles are going when they hit the window (v)



Approach:
Step 1: Choose coordinate system.
Step 2: You know it is a projectile so write down the basic equations of motion and integrate them so you have a description of how the “pebbles” will move.
Step 3: Carefully think about what you know and don’t know.
Step 4: Make sure you have equal numbers of equations and unknowns.
Step 5: Solve the equations.



Solution:
Step 1 & 2:

$$\ddot{x} = a_x = 0$$
$$\dot{x} = v_{0x}$$
$$x = v_{0x}t + x_0$$

$$\ddot{y} = a_y = -g$$
$$\dot{y} = -gt + v_{0y}$$
$$y = -g\frac{t^2}{2} + v_{0y}t + y_0$$

Step 3: What do we know?

1. *H*=maximum height – Why?
2. *D*=distance traveled – What is it?

What is unknown?

1. Initial speed v_0
2. Initial direction θ
3. Time to reach the window t_w

Step 4: There are three unknowns. What equations can we write?

$$x(t_w) = v_{0x} t_w = D$$

$$v_0 \cos\theta t_w = D$$

$$y(t_w) = -g\frac{t_w^2}{2} + v_{0y} t_w = H$$

$$-g\frac{t_w^2}{2} + v_0 \sin\theta t_w = H$$

That's two equations…oh, oh … what else is there?

What about….

$$\dot{y}(t_w) = -gt_w + v_0 \sin\theta = 0$$

So we have three equations and three unknowns…now we need to go about solving them.

Step 5: Start with the last equation above….

$$-gt_w + v_0 \sin\theta = 0$$

$$\sin\theta = \frac{gt_w}{v_0}$$

Put it into the second equation and the initial speed and angle cancel out leaving only the time.

$$-g\frac{t_w^2}{2} + v_0 \sin\theta t_w = H$$

$$-g\frac{t_w^2}{2} + v_0 \frac{gt_w}{v_0} t_w = H$$

$$-g\frac{t_w^2}{2} + gt_w^2 = H$$

$$\frac{gt_w^2}{2} = H$$

$$t_w = \sqrt{\frac{2H}{g}}$$

Now when you apply math to engineering problems the solutions and even the way the equations look means something. What does this equation mean? What can get the time from H alone? Can we get the time from D alone too?

Now the "boring part"…plug in the numbers…

$$t_w = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(8.0m)}{9.81m/s^2}} = 1.27\sec$$

$$v_0 \cos\theta = \frac{D}{t_w} \quad + \quad v_0 \sin\theta = g t_w \quad \Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{t_w^2 g}{D} \quad \Rightarrow \theta = 60.6^o$$

$$v_0 = \frac{g t_w}{\sin\theta} = 14.4 m/s \qquad v_{0y} = v_0 \sin\theta = 12.5 \qquad v_{0x} = v_0 \cos\theta = 7.0$$

Line of sight versus launch angle…what does that mean?