Lecture 3 – Rectilinear Motion of Particles, Example Problems

Administrative Issues

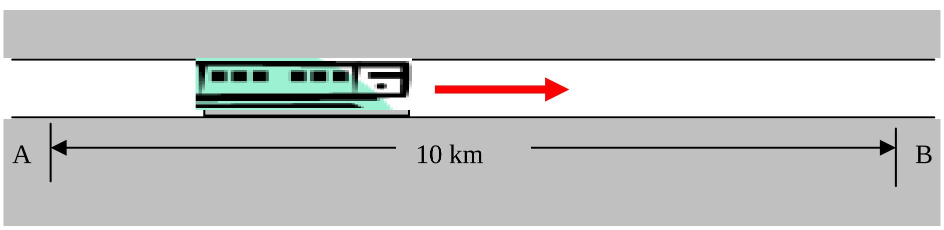
- Homework 1 Due Monday 8/29
- Homework 2 Download from WEBCT , Due Friday 9/2
- Chatroom Help, Thurs 9 PM (have to put our baby to sleep first!) , Go to WEBCT Chatroom, and go to Room 1

Introduction:

Today, we are going to do some example problems to illustrate the concepts we discussed last time and hopefully help you with homework 2.

Example 1

A vacuum-propelled capsule for a high-speed tube transportation system of the future is being designed for operation between two stations A and B, which are 10 km apart. If the acceleration and deceleration are to have a limiting magnitude of 0.6g and if the velocities are to be limited to 400 km/hr, determine the minimum time t for the capsule to make the 10-km trip.



Solution

The fastest way for the train to make the 10 km trip is to accelerate at maximum capability to maximum velocity, remain at maximum velocity as long as possible and the decelerate at maximum capability down to zero.

Stage 1: Initial Acceleration

$$a_0 = 0.6g = 5.88\, m/s^2$$

$$v = a_0 t + v_0 = a_0 t$$

$$x = \frac{1}{2} a_0 t^2 + x_0 = \frac{1}{2} a_0 t^2$$

The time for the velocity to reach its maximum velocity and the position of the train at that time is given by,

$$a_0 t_1 = v_{\max} = 400000\, m/hr = 111 m/s$$
$$t_1 = 18.9\, s$$
$$x_1 = \frac{1}{2} a_0 t_1^2 = 1049\, m$$

Stage 2: Constant Velocity Phase
The deceleration at the end of the trip will be of the same distance and duration as the acceleration phase. Thus, the distance traveled at constant velocity will be given by,

$$d_{cv} = 10000\, m - 2x_1 = 7903 m$$
$$t_{cv} = \frac{d_{cv}}{111.1 m/s} = 71.13\ s$$

Stage 3: Constant deceleration
The length of the deceleration is the same as the acceleration phase, 18.9 s.

The total time is the sum of the three phases 108.9 sec (1.81 min).

Example 2: Problem 2/48

To a close approximation, the pressure behind a rifle bullet varies inversely with the position *x* of the bullet along the barrel. Thus, the acceleration of the bullet may be written as *a=k/x* where *k* is a constant. If the bullet starts from rest at x=7.5 mm and if the muzzle velocity of the bullet is 600 m/s at the end of the 750 mm at the end of the barrel, compute the acceleration of the bullet as it passes the midpoint of the barrel at x=375 mm.

Approach

Step 1: Write the general relation between the velocity to the position in the case of a position dependent acceleration…see the important mathematical trick form the last lecture.

Step 2: Combine the relation between Step 1 and the specific expression for the acceleration.

Step 3: Complete the integral.

Step 4: Based on the information given solve for the constant *k*.

Step 5: Find the acceleration at the midpoint using the value of *k*.

Solution

Step 1: The general relation is as follows.

$$\frac{a}{v} = \frac{dv/dt}{dx/dt} = \frac{dv}{dx}$$ ← **Important**

Step 2: Combine.

$$\frac{dv}{dx} = \frac{a}{v}$$

$$\frac{dv}{dx} = \frac{k/x}{v}$$

Step 3: Separate variables and integrate.

$$vdv = \frac{k}{x}dx$$

Now integrate and make sure to include a nonzero initial position.

$$\int_0^v vdv = \int_{x_o}^x \frac{k}{x}dx$$

Note that on the velocity limits are form zero to a final velocity *v* and the *x* limits are form x_0, the initial position to a final position *x*. This allows us to write the velocity as a function of the position.

$$\frac{v^2}{2} = k \ln x \Big|_{x_0}^{x}$$

$$\frac{v^2}{2} = k \ln \frac{x}{x_0}$$

Step 4: Using the data from above, we find *k*.

$$k = \frac{v^2}{2 \ln \frac{x}{x_0}}$$

$$k = \frac{(600 m/s)^2}{2 \ln \frac{750}{7.5}}$$

$$k = 39087 m^2/s^2$$

Step 5: Now the acceleration at the middle of the barrel is found by putting in the desired *x* value.

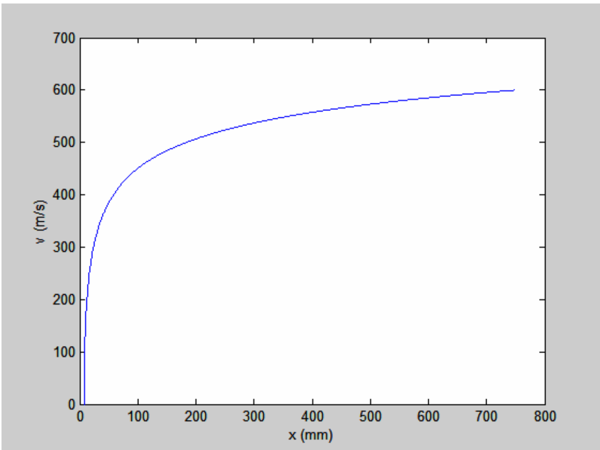
$$a = \frac{k}{x}$$

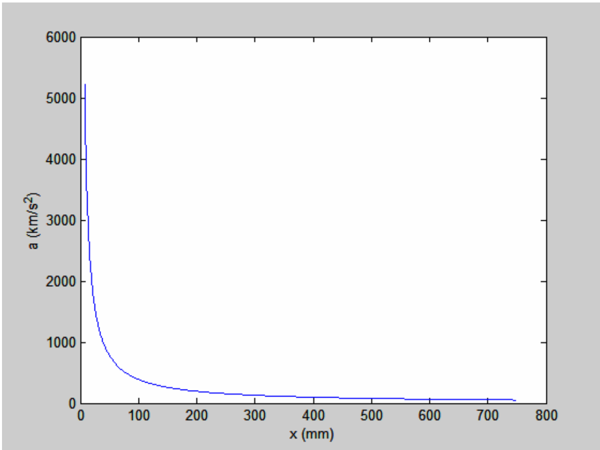
$$a = \frac{39087 m^2/s^2}{0.375 m}$$

$$a = 104\,230 m/s^2 (10\,000 g's!!!)$$

```
% Problem 2_48

% Constant k
k=3.9087e+004;

% Position Vector
x=[7.5:0.5:750];
x0=7.5;

% Calcuation of Velocities and
Accelerations
a=k./x;
v=sqrt(2*k*log(x./x0))

% Plot the Acceleration and Velocity
figure(1)
plot(x,v);
xlabel('x (mm)')
ylabel('v (m/s)')

figure(2)
plot(x,a);
xlabel('x (mm)')
ylabel('a (km/s^2)')
```

It is interesting to look at the velocity and acceleration as a function of position. We use MATLAB to plot these. The MATLAB code is given also.





Notice that the acceleration is extremely high in the beginning and corresponding to this the velocity increases very rapidly. As the bullet moves down the barrel, the rate of increase in velocity decreases. Notice that by 375mm, the acceleration has dropped to 104 km/s2 as we calculated in Step 5.